DABOLL'S COMPLETE

SCHOOLMASTER'S ASSISTANT:

BEING A

PLAIN COMPREHENSIVE

SYSTEM OF PRACTICAL

ARITHMETIC,

ADAPTED TO THE USE OF SCHOOLS

IN THE

UNITED STATES:

EXEMPLIFIED AND ILLUSTRATED IN A MANNER CALCULATED TO ENGAGE THE MINDS OF YOUTH IN THEIR STUDY, AND IMPART TO THEM A THOROUGH KNOWLEDGE OF PRACTICAL

ARITHMETIC.

BY NATHAN DABOLL, A.M. AND
DAVID A. DABOLL.

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PREFACE.

This work is designed to furnish Schools with a methodical, comprehensive and practical system of Arithmetic; in which it has been the study of the Authors to render it a useful and easy text-book for Instructors, and also to convey instruction in an easy, concise and familiar manner to the pupil. The Authors are not disposed to derogate the merits of any system of Arithmetic now before the public, of which there is a great variety, and many meritorious works; none of which, perhaps, has received a greater share of public patronage than Daboll's Schoolmaster's Assistant: But it must be obvious to teachers of Arithmetic, and also to men of business, that Arithmetics written some thirty or forty years ago, however complete they were considered at that time, are now become more or less obsolete. One objection to those which were written about the time Federal Money was coming into use, is, that they are too much intermixed with, and contain more of the old currency or Sterling Money, than is necessary at the present day. Another objection to those of long standing is, a deficiency of proper illustrations of the rules and examples.

To remedy these defects, a variety of Arithmetics have been compiled by different authors and published within a few years past. But for the use of Schools generally, there are in the opinion of experienced teachers some objections to them all: for while the older publications have pursued too much of an arbitrary, dogmatic course of instruction; other more modern writers have pursued almost wholly the inductive or mental plan, leaving the pupil to solve without proper and concise rules nearly all questions in Arithmetic; and thus in endeavoring to correct one error, have, as regards *Common Schools*, run into another as great. A small introduction to the ground rules of Arithmetic, on the inductive or mental plan, may be useful: but considering that our Common Schools are made up principally of the children of mechanics, farmers, and working-men, whose time, allotted them to obtain a knowledge of Arithmetic and other branches of literature, is but a few months in each year, for a short period of years; to require them to perform the various operations in Arithmetic without plain, concise rules and illustrations, is as inconsistent as it would be to require each laboring man in the community, to make every implement of his particular trade, before he could enter upon the business of his occupation.

In the execution and arrangement of "DABOLL'S COMPLETE SCHOOLMASTER'S ASSISTANT," now offered to the public, it has been the object of the Authors alike to guard against the dogmatic course pursued in Arithmetics of long standing, and also the deficiency of concise rules, so apparent in some of the more modern ones. In this work the Authors have given a short introduction to the first rules, on the inductive or mental plan of instruction, after which, it has been their object to give the rules, examples and illustrations, in a manner so clear and familiar as to be easily comprehended by the pupils, and also, to convey to their minds the reason of the same.

They have also pursued a course of questioning on the rules, which will be found beneficial to the pupil, and convenient to the instructor. It is not the design of this work, to call forth the deep research of men of science, but

rather to develope to the juvenile mind, a plain, easy and pleasing ascent, in the science of practical Arithmetic.

The arrangement of the rules and examples in this work is such as is believed to be the most proper.

Addition and Subtraction of Federal Money are placed immediately after Addition and Subtraction of whole numbers.

Reduction, Multiplication and Division of Federal Money, with simple and concise rules for finding the cost of goods, &c. when the price is an aliquot or even part of a dollar, are placed immediately after Division of whole numbers.

In Reduction Ascending and Descending, the answers to the questions are not set down, as they alternately prove each other.

Fractions, Vulgar and Decimal, have received that attention which their importance demands; being simplified and illustrated in such a manner as to render the study of them pleasing and interesting to the pupil.

In Simple Interest several short rules are given. Also explanatory observations, and remarks on casting Interest on notes, bonds, &c. where endorsements have been made.

The rule called Practice is omitted, except so much as is now necessary in business.

That part of the rule formerly called "Tare and Tret" which relates to trett, cloff, and suttle, is omitted, it being entirely obsolete.

A short demonstration of the Square and Cube Roots is given; and the rules for working Arithmetical and Geometrical Progression, will be found very plain and concise.

After going through the various rules, a collection of useful and entertaining questions is given for exercise.

The Appendix contains a variety of useful Problems in Mensuration, &c. Also a concise method of BOOK-KEEPING, adapted to the Business of Farmers, Mechanics, &c.

Some of the late writers on Arithmetic, have wholly expunged the old currency of Sterling Money; but considering the increasing facilities to commerce, and the contiguity of these States to the British dominions; the Authors have thought proper to retain enough of Sterling Money, to show its use and nature.

TABLE OF CONTENTS.

ARITHMETICAL TABLES.

ADDITION TABLE

2 and 1 are 3	3 and 1 are 4	4 and 1 are 5	5 and 1 are 6
2 and 2 are 4	3 and 2 are 5	4 and 2 are 6	5 and 2 are 7
2 and 3 are 5	3 and 3 are 6	4 and 3 are 7	5 and 3 are 8
2 and 4 are 6	3 and 4 are 7	4 and 4 are 8	5 and 4 are 9
2 and 5 are 7	3 and 5 are 8	4 and 5 are 9	5 and 5 are 10
2 and 6 are 8	3 and 6 are 9	4 and 6 are 10	5 and 6 are 11
2 and 7 are 9	3 and 7 are 10	4 and 7 are 11	5 and 7 are 12
2 and 8 are 10	3 and 8 are 11	4 and 8 are 12	5 and 8 are 13
2 and 9 are 11	3 and 9 are 12	4 and 9 are 13	5 and 9 are 14
2 and 10 are 12	3 and 10 are 13	4 and 10 are 14	5 and 10 are 15
2 and 11 are 13	3 and 11 are 14	4 and 11 are 15	5 and 11 are 16
2 and 12 are 14	3 and 12 are 15	4 and 12 are 16	5 and 12 are 17
6 and 1 are 7	7 and 1 are 8	8 and 1 are 9	9 and 1 are 10
6 and 2 are 8	7 and 2 are 9	8 and 2 are 10	9 and 2 are 11
6 and 3 are 9	7 and 3 are 10	8 and 3 are 11	9 and 3 are 12
6 and 4 are 10	7 and 4 are 11	8 and 4 are 12	9 and 4 are 13
6 and 5 are 11	7 and 5 are 12	8 and 5 are 13	9 and 5 are 14
6 and 6 are 12	7 and 6 are 13	8 and 6 are 14	9 and 6 are 15
6 and 7 are 13	7 and 7 are 14	8 and 7 are 15	9 and 7 are 16
6 and 8 are 14	7 and 8 are 15	8 and 8 are 16	9 and 8 are 17
6 and 9 are 15	7 and 9 are 16	8 and 9 are 17	9 and 9 are 18
6 and 10 are 16	7 and 10 are 17	8 and 10 are 18	9 and 10 are 19
6 and 11 are 17	7 and 11 are 18	8 and 11 are 19	9 and 11 are 20
6 and 12 are 18	7 and 12 are 19	8 and 12 are 20	9 and 12 are 21
10 and 1 are 11	10 and 10 are 20	11 and 7 are 18	12 and 4 are 16
10 and 2 are 12	10 and 11 are 21	11 and 8 are 19	12 and 5 are 17
10 and 3 are 13	10 and 12 are 22	11 and 9 are 20	12 and 6 are 18
10 and 4 are 14	11 and 1 are 12	11 and 10 are 21	12 and 7 are 19
10 and 5 are 15	11 and 2 are 13	11 and 11 are 22	12 and 8 are 20
10 and 6 are 16	11 and 3 are 14	11 and 12 are 23	12 and 9 are 21
10 and 7 are 17	11 and 4 are 15	12 and 1 are 13	12 and 10 are 22
10 and 8 are 18	11 and 5 are 16	12 and 2 are 14	12 and 11 are 23
10 and 9 are 19	11 and 6 are 17	12 and 3 are 15	12 and 12 are 24

Signs. A cross + with one line perpendicular and the other horizontal, is the sign of addition. It shows that the numbers between which it is placed are to be added together. It is sometimes read *plus*, which is a Latin word signifying *more*. It also denotes a remainder after division.

Two horizontal parallel lines =, are the sign of equality. It signifies that the number before it is equal to the number after it. Thus, 100 cents=1 dollar; read, 100 cents are

equal to 1 dollar. $5+7=12$; read 5 and 7 added together are 12.

Questions.

8+3=how many ?	6+8=how many ?	8+12=how many ?
7+6=how many ?	3+6=how many ?	5+12=how many ?
9+4=how many ?	4+7=how many ?	11+10=how many ?
5+6=how many ?	5+8=how many ?	2+11=how many ?
7+2=how many ?	6+5=how many ?	4+11=how many ?
11+7=how many ?	7+9=how many ?	6+ 7=how many ?
12+6=how many ?	6+7=how many ?	10+ 7=how many ?
10+9=how many ?	9+6=how many ?	4+11=how many ?

SUBTRACTION TABLE.

2 from 2 leave 0	3 from 3 leave 0	4 from 4 leave 0
2 from 3 leave 1	3 from 4 leave 1	4 from 5 leave 1
2 from 4 leave 2	3 from 5 leave 2	4 from 6 leave 2
2 from 5 leave 3	3 from 6 leave 3	4 from 7 leave 3
2 from 6 leave 4	3 from 7 leave 4	4 from 8 leave 4
2 from 7 leave 5	3 from 8 leave 5	4 from 9 leave 5
2 from 8 leave 6	3 from 9 leave 6	4 from 10 leave 6
2 from 9 leave 7	3 from 10 leave 7	4 from 11 leave 7
2 from 10 leave 8	3 from 11 leave 8	4 from 12 leave 8
2 from 11 leave 9	3 from 12 leave 9	4 from 13 leave 9
5 from 5 leave 0	6 from 6 leave 0	7 from 7 leave 0
5 from 6 leave 1	6 from 7 leave 1	7 from 8 leave 1
5 from 7 leave 2	6 from 8 leave 2	7 from 9 leave 2
5 from 8 leave 3	6 from 9 leave 3	7 from 10 leave 3
5 from 9 leave 4	6 from 10 leave 4	7 from 11 leave 4
5 from 10 leave 5	6 from 11 leave 5	7 from 12 leave 5
5 from 11 leave 6	6 from 12 leave 6	7 from 13 leave 6
5 from 12 leave 7	6 from 13 leave 7	7 from 14 leave 7
5 from 13 leave 8	6 from 14 leave 8	7 from 15 leave 8
5 from 14 leave 9	6 from 15 leave 9	7 from 16 leave 9
8 from 8 leave 0	9 from 9 leave 0	10 from 10 leave 0
8 from 9 leave 1	9 from 10 leave 1	10 from 11 leave 1
8 from 10 leave 2	9 from 11 leave 2	10 from 12 leave 2
8 from 11 leave 3	9 from 12 leave 3	10 from 13 leave 3
8 from 12 leave 4	9 from 13 leave 4	10 from 14 leave 4
8 from 13 leave 5	9 from 14 leave 5	10 from 15 leave 5
8 from 14 leave 6	9 from 15 leave 6	10 from 16 leave 6
8 from 15 leave 7	9 from 16 leave 7	10 from 17 leave 7
8 from 16 leave 8	9 from 17 leave 8	10 from 18 leave 8
8 from 17 leave 9	9 from 18 leave 9	10 from 19 leave 9

Sign. A short horizontal line —, is the sign of subtraction; it is usually read *minus*, which is a Latin word signifying *less*. It shows that the number after it is to be taken from the number before it; thus, $6-2=4$, read 6 less 2 is equal to 4, or 2 subtracted from 6 leaves 4.

Questions.

6—2=how many?	10— 7=how many?	18—9=how many?
7—4=how many?	13— 8=how many?	16—7=how many?
9—5=how many?	15— 9=how many?	11—5=how many?
9—6=how many?	11— 6=how many?	9—4=how many?
5—3=how many?	13— 9=how many?	7—3=how many?
8—4=how many?	15—10=how many?	12—5=how many?

MULTIPLICATION TABLE

2 times 1 are 2	3 times 1 are 3	4 times 1 are 4
2 times 2 are 4	3 times 2 are 6	4 times 2 are 8
2 times 3 are 6	3 times 3 are 9	4 times 3 are 12
2 times 4 are 8	3 times 4 are 12	4 times 4 are 16
2 times 5 are 10	3 times 5 are 15	4 times 5 are 20
2 times 6 are 12	3 times 6 are 18	4 times 6 are 24
2 times 7 are 14	3 times 7 are 21	4 times 7 are 28
2 times 8 are 16	3 times 8 are 24	4 times 8 are 32
2 times 9 are 18	3 times 9 are 27	4 times 9 are 36
2 times 10 are 20	3 times 10 are 30	4 times 10 are 40
2 times 11 are 22	3 times 11 are 33	4 times 11 are 44
2 times 12 are 24	3 times 12 are 36	4 times 12 are 48
5 times 1 are 5	6 times 1 are 6	7 times 1 are 7
5 times 2 are 10	6 times 2 are 12	7 times 2 are 14
5 times 3 are 15	6 times 3 are 18	7 times 3 are 21
5 times 4 are 20	6 times 4 are 24	7 times 4 are 28
5 times 5 are 25	6 times 5 are 30	7 times 5 are 35
5 times 6 are 30	6 times 6 are 36	7 times 6 are 42
5 times 7 are 35	6 times 7 are 42	7 times 7 are 49
5 times 8 are 40	6 times 8 are 48	7 times 8 are 56
5 times 9 are 45	6 times 9 are 54	7 times 9 are 63
5 times 10 are 50	6 times 10 are 60	7 times 10 are 70
5 times 11 are 55	6 times 11 are 66	7 times 11 are 77
5 times 12 are 60	6 times 12 are 72	7 times 12 are 84
8 times 1 are 8	9 times 1 are 9	10 times 1 are 10
8 times 2 are 16	9 times 2 are 18	10 times 2 are 20
8 times 3 are 24	9 times 3 are 27	10 times 3 are 30
8 times 4 are 32	9 times 4 are 36	10 times 4 are 40
8 times 5 are 40	9 times 5 are 45	10 times 5 are 50
8 times 6 are 48	9 times 6 are 54	10 times 6 are 60
8 times 7 are 56	9 times 7 are 63	10 times 7 are 70
8 times 8 are 64	9 times 8 are 72	10 times 8 are 80
8 times 9 are 72	9 times 9 are 81	10 times 9 are 90
8 times 10 are 80	9 times 10 are 90	10 times 10 are 100
8 times 11 are 88	9 times 11 are 99	10 times 11 are 110
8 times 12 are 96	9 times 12 are 108	10 times 12 are 120
11 times 1 are 11	11 times 9 are 99	12 times 5 are 60
11 times 2 are 22	11 times 10 are 110	12 times 6 are 72
11 times 3 are 33	11 times 11 are 121	12 times 7 are 84
11 times 4 are 44	11 times 12 are 132	12 times 8 are 96
11 times 5 are 55	12 times 1 are 12	12 times 9 are 108
11 times 6 are 66	12 times 2 are 24	12 times 10 are 120
11 times 7 are 77	12 times 3 are 36	12 times 11 are 132
11 times 8 are 88	12 times 4 are 48	12 times 12 are 144

Sign. Two short lines crossing each other in the form of the letter X, are the sign of multiplication ; thus, $4\times3=12$, signifies that 4 multiplied by 3 is equal to 12.

Questions.

8×2=how many ?	2× 8=how many ?	10× 8=how many ?
7×3=how many ?	8× 6=how many ?	12× 9=how many ?
5×4=how many ?	5× 3=how many ?	12× 7=how many ?
6×5=how many ?	4× 7=how many ?	11× 6=how many ?
9×6=how many ?	8× 6=how many ?	10× 6=how many ?
5×3=how many ?	9× 3=how many ?	12×11=how many ?
4×6=how many ?	7×10=how many ?	12×12=how many ?

DIVISION TABLE.

2 in 2, 1 time	3 in 3, 1 time	4 in 4, 1 time	5 in 5, 1 time
2 in 4, 2 times	3 in 6, 2 times	4 in 8, 2 times	5 in 10, 2 times
2 in 6, 3 times	3 in 9, 3 times	4 in 12, 3 times	5 in 15, 3 times
2 in 8, 4 times	3 in 12, 4 times	4 in 16, 4 times	5 in 20, 4 times
2 in 10, 5 times	3 in 15, 5 times	4 in 20, 5 times	5 in 25, 5 times
2 in 12, 6 times	3 in 18, 6 times	4 in 24, 6 times	5 in 30, 6 times
2 in 14, 7 times	3 in 21, 7 times	4 in 28, 7 times	5 in 35, 7 times
2 in 16, 8 times	3 in 24, 8 times	4 in 32, 8 times	5 in 40, 8 times
2 in 18, 9 times	3 in 27, 9 times	4 in 36, 9 times	5 in 45, 9 times
6 in 6, 1 time	7 in 7, 1 time	8 in 8, 1 time	9 in 9, 1 time
6 in 12, 2 times	7 in 14, 2 times	8 in 16, 2 times	9 in 18, 2 times
6 in 18, 3 times	7 in 21, 3 times	8 in 24, 3 times	9 in 27, 3 times
6 in 24, 4 times	7 in 28, 4 times	8 in 32, 4 times	9 in 36, 4 times
6 in 30, 5 times	7 in 35, 5 times	8 in 40, 5 times	9 in 45, 5 times
6 in 36, 6 times	7 in 42, 6 times	8 in 48, 6 times	9 in 54, 6 times
6 in 42, 7 times	7 in 49, 7 times	8 in 56, 7 times	9 in 63, 7 times
6 in 48, 8 times	7 in 56, 8 times	8 in 64, 8 times	9 in 72, 8 times
6 in 54, 9 times	7 in 63, 9 times	8 in 72, 9 times	9 in 81, 9 times

10 in 10, 1 time	11 in 11, 1 time	12 in 12, 1 time
10 in 20, 2 times	11 in 22, 2 times	12 in 24, 2 times
10 in 30, 3 times	11 in 33, 3 times	12 in 36, 3 times
10 in 40, 4 times	11 in 44, 4 times	12 in 48, 4 times
10 in 50, 5 times	11 in 55, 5 times	12 in 60, 5 times
10 in 60, 6 times	11 in 66, 6 times	12 in 72, 6 times
10 in 70, 7 times	11 in 77, 7 times	12 in 84, 7 times
10 in 80, 8 times	11 in 88, 8 times	12 in 96, 8 times
10 in 90, 9 times	11 in 99, 9 times	12 in 108, 9 times

Signs. A horizontal line with a dot above and below it, $\div$, is the sign of division. It shows that the number *before* it is to be divided by the number *after* it ; thus, $12\div4=3$ signifies that 12 divided by 4 is equal to 3. Also, the divisor placed under the dividend with a line between, $\frac{12}{4}$, is the sign of division.

Questions.

4÷2=how many ?	77÷11=how many ?	$\frac{49}{7}$=how many ?
9÷3=how many ?	35÷ 7=how many ?	$\frac{54}{6}$=how many ?
84÷7=how many ?	66÷ 8=how many ?	$\frac{72}{8}$=how many ?
42÷6=how many ?	84÷12=how many ?	$\frac{36}{6}$=how many ?
54÷9=how many ?	99÷11=how many ?	
63÷9=how many ?	81÷ 9=how many ?	
32÷8=how many ?	108÷12=how many ?	

Note. The scholar should perfectly understand the signs.

TABLES OF COMPOUND NUMBERS.

Troy Weight.

By Troy weight are weighed Gold,* Silver, Jewels, and all liquors.

24 grains (grs.)	make 1	pennyweight,	marked	pwt.
20 pwts.	make 1	ounce,	"	oz.
12 ounces	make 1	pound,	"	lb.

Apothecaries Weight.

Apothecaries weight is used in compounding medicines, but not in selling them.

20 grains (grs.)	make 1	scruple,	marked	℈
3 scruples	make 1	dram,	"	ʒ
8 drams	make 1	ounce,	"	℥
12 ounces	make 1	pound,	"	lb.

Avoirdupoise Weight.

Avoirdupoise weight is the common weight, used in weighing all coarse and drossy goods, grocery-wares, and all metals except gold and silver.

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* The fineness of Gold is tried by fire, and is reckoned in carats, by which is understood the 24th part of any quantity. If it lose nothing in the trial, it is called 24 carats fine: if it lose 2 carats, it is called 22 carats fine, which is the standard of gold coin.

Silver which loses nothing in trial, is accounted 12 ounces fine. The standard for silver coin is 11 oz. 2 pwts. of fine silver and 18 pwts. of copper melted together

16 drams (dr.)	make 1 ounce,	marked	oz.
16 ounces	make 1 pound	"	lb.
28 pounds	make 1 quarter of a hund. weight,		qr.
4 quarters	make 1 hundred weight,	marked	cwt.
20 cwt.	make 1 Ton,	"	T.

Note. By this Table it will be perceived that a hundred weight is 112 pounds; but in many places, especially in seaports, traders buy and sell by the 100 lbs. which is also established by law in several states.

Cloth Measure.

Cloth measure is used in measuring all kinds of cloth, lace, &c.

$2\frac{1}{4}$ inches (in.)	make 1 nail,	marked	na.
4 nails	make 1 quarter of a yard,	"	qr.
4 quarters	make 1 yard,	"	yd.
3 quarters	make 1 Ell Flemish,	"	E.Fl.
5 quarters	make 1 Ell English,	"	E.E.
6 quarters	make 1 Ell French,	"	E.F.

Dry Measure.

Dry measure is used in measuring grain, salt, fruit, coal, roots, seeds, oysters, &c.

2 pints (pt.)	make 1 quart,	marked	qt.
8 quarts	make 1 peck,	"	pk.
4 pecks	make 1 bushel,	"	bu.

Note. A gallon, dry measure, contains $268\frac{4}{5}$ cubic inches; a bushel contains $2150\frac{2}{5}$ cubic inches.

Wine Measure.

Wine measure is used in measuring all spirituous liquors, molasses, vinegar, oil, &c. 231 cubic inches make a gallon.

4 gills (gi.)	make 1 pint,	marked	pt.
2 pints	make 1 quart,	"	qt.
4 quarts	make 1 gallon,	"	gal.
$31\frac{1}{2}$ gallons	make 1 barrel,	"	bl.
42 gallons	make 1 tierce,	"	tier.
63 gallons	make 1 hogshead,	"	hhd.
2 hogsheads	make 1 pipe,	"	P.
2 pipes	make 1 ton,	"	T.

Long Measure

Long measure is used in measuring distances, or other things where length is considered, without regard to breadth.

3 barley-corns (b.c.)	make	1	inch,	marked in.
12 inches	make	1	foot,	" ft.
3 feet	make	1	yard,	marked yd.
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	make	1	rod, pole, or perch,	rd.
40 rods	make	1	furlong,	fur.
8 furlongs or 320 rds.	make	1	mile,	M.
3 miles	make	1	league,	L.
$69\frac{1}{2}$ statute miles	make	1	degree,	°

360 degrees the circumference of the earth.

Note. In measuring the height of horses, 4 inches make 1 hand. In measuring depths, 6 feet make 1 fathom.

Land or Square Measure.

Square measure is used in measuring land and any other thing where length and breadth only are considered.

144 square inches make 1 square foot.
9 square feet make 1 square yard.
$30\frac{1}{4}$ square yards or $272\frac{1}{4}$ square feet make 1 square rod.
40 square rods make 1 rood or quarter of an acre.
4 roods or 160 square rods make 1 acre.
640 square acres make 1 square mile.

Note. Gunter's Chain, used in measuring distances, is 4 rods in length, containing 100 links, each link being $7\frac{92}{100}$ inches in length. 25 links make 1 rod.

Solid or Cubic Measure.

Solid or cubic measure is used in measuring things that have length, breadth, and thickness.

1728 solid inches	make 1 solid foot.
40 feet of round timber, or 50 feet of hewn timber,	make 1 ton or load.
27 solid feet	make 1 solid yard.
128 solid feet, or 8 feet long × 4 wide and × 4 high,	make 1 cord of wood.

Time.

Time is naturally divided into years, by the revolution of the earth round the sun; and into days, by the revolution of the earth upon its own axis.

60	seconds (s.)	make 1	minute,	marked	m.
60	minutes	make 1	hour,	"	h.
24	hours	make 1	day,	"	d.
365	days	make 1	year,	"	yr.
	—ALSO—				
7	days	make 1	week,	"	w.
4	weeks	make 1	month,	"	mo.
13	mos. 1 d. and 6 hours	make 1	Julian year.		
12	Calendar months	make 1	year,	"	yr.

Thirty days hath September,
April, June, and November,
February twenty-eight alone,
All the rest have thirty-one.

Bissextile or leap-year comes once in 4 years, in which February hath 29 days. *Note.* When a year can be divided by 4 without a remainder, it is leap-year.

Circular Motion.

Circular motion is the motion of the earth and other planets round the sun; and is applied to latitude and longitude.

60	seconds (″)	make 1	minute,	marked	′
60	minutes	make 1	degree,	"	°
30	degrees	make 1	sign,	"	S

12 signs, or 360 degrees, the whole great circle of the Zodiac.

Denominations of things not included in the foregoing Tables.

12	single things	make 1	dozen.
12	dozen	make 1	gross.
12	gross, or 144 dozen,	make 1	great gross.
20	single things	make 1	score.
5	score	make 1	hundred.
200	lbs. of Beef or Pork	make 1	barrel.
112	lbs. of Fish	make 1	quintal.
24	sheets of Paper	make 1	quire.
20	quires	make 1	ream.

ARITHMETIC.

1 ARITHMETIC is the art, or science, of computing by numbers. It has five principal Rules for its operation, viz.: *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*.

NUMERATION.

2. Numeration teaches how to read and write numbers.

3. A single or individual thing is called a unit.

4. The following are the ten characters used in computation.

A unit or one, written		1
Two,	"	2
Three,	"	3
Four,	"	4
Five,	"	5
Six,	"	6
Seven,	"	7
Eight,	"	8
Nine,	"	9
Cipher,	"	0

5. The nine first of these characters are called significant figures or digits; and have each of them a simple and a local value.

6. When standing separately or alone they express; or represent their simple value only; viz. one, two, three, four, five, &c.

7. But when placed or located with other figures, they have a local value, according to the place they stand in; counting from the right hand towards the left. For example, the number one hundred and eleven is made by repeating the figure 1 three times.

Hunds.	Tens.	Units.
1	1	1

Thus, 1 1 1. The 1 on the right hand in the unit's place represents its simple value of one only. The same figure located in the second place towards the left hand, is increased in its value ten times, and now counts one ten, because it stands in the place of tens: and the same figure

again located or placed in the third place towards the left, is increased in its value one hundred times, and now counts one hundred, because it stands in the place of hundreds. And one hundred, one ten, and one unit, make one hundred and eleven.

8. Hence it appears that any figure in the unit's place, expresses its simple value only, or so many ones; but in the second place, or place of tens, it becomes so many tens, or ten times its simple value; and in the third place, or place of hundreds it becomes a hundred times its simple value, and so on, as in the following

Numeration Table.

Hundreds of millions,	Tens of millions,	Millions,	Hundreds of thousands,	Tens of thousands,	Thousands,	Hundreds,	Tens,	Units,	
								1	—One unit.
							2	1	—2 tens and 1 unit, or twenty-one.
						3	2	1	3 hundreds, 2 tens and 1 unit, or three hundred and twenty-one.
					4	3	2	1	4 thous. 3 hunds. two tens, one unit, or four thous, three hund. and twenty-one.
				5	4	3	2	1	5 tens of thous. 4 thousands, or fifty-four thous. three hund. and twenty-one.
			6	5	4	0	0	0	6 hund. of thous. five tens of thous. four thous., or six hund. and fifty-four thous.
		7	6	5	4	3	2	1	7 millions 6 hundred and fifty-four thousand three hundred and twenty-one.
	8	7	0	0	0	0	0	0	8 tens of millions, 7 millions, or eighty-seven millions.
9	8	7	0	0	0	0	0	0	9 hund. of millions, 8 tens of millions, 7 millions, or 987 millions.
1	2	3	4	5	6	7	8	9	—123 millions 456 thousand 789.
8	7	9	6	4	5	3	8	4	—879 millions 645 thousand 384.

Note.—There are two methods of expressing numbers shorter than writing them in words, viz: The method of expressing numbers by letters, called the Roman method of computation, which was invented and used by the Romans. But at this day it is seldom used except in numbering chapters in books, &c.

The method of computing by figures, as above, was invented and used by the Arabs, and afterwards was introduced into Europe, and thence into this country, and is now in general use in all parts of the civilized world. This is called the Arabic method of computation, because it was invented by the Arabs.

9. The cipher, when standing alone, signifies nothing; but when placed on the right hand of the significant figures it increases their value in a tenfold proportion.—Thus 1, with a cipher annexed to it, becomes 10, ten, because 1 is thereby removed into the tens' place, and with two ciphers annexed, it becomes 100, one hundred, the 1 being now removed to the place of hundreds, &c.

10. *To read numbers, or to know the value of any number of figures.*

RULE. 1. Begin at the right hand and numerate towards the left, by saying units, tens, hundreds, thousands, &c., as in the Numeration Table.

2. Then to the simple value of each figure join the name of its place, beginning at the left hand and reading to the right.

EXAMPLES.

Read the following numbers.

124	One hundred and twenty-four.
365	Three hundred and sixty-five.
4628	Four thousand six hundred and twenty-eight.
54026	Fifty four thousand and twenty-six.
144321	One hundred and forty-four thousand three hundred and twenty-one.
5684568	Five millions six hundred eighty-four thousand five hundred and sixty-eight.

For convenience in reading large numbers, it is the usual method to point them off into periods of three figures each, as follows:

879	Eight hundred seventy-nine.
879 000	Eight hundred seventy-nine thousand.
879 000 000	Eight hundred seventy-nine millions.
987 854 321	Nine hundred and eighty-seven millions, eight hundred and fifty-four thousand, three hundred and twenty-one.

11. *To write numbers.*

RULE. 1. Begin at the right hand and write the numbers according to their proper value in numeration: that is, write units in the place of units, tens in the place of tens, &c.

2. Observe always to supply those places with ciphers, which are omitted in the question, in their proper order.

EXAMPLES.

Write, in figures, the following numbers.

1. Two hundred and five.

We begin at the right hand and write units in the place of units; thus, 5; there are no tens, therefore we supply the place with a cipher, 05; next we write down the hundreds, 205.

2. Thirty-three.
3. Eight hundred and thirty-three.
4. One thousand eight hundred and thirty-three
5. Forty-five thousand six hundred.
6. Three hundred and forty-five thousand six hundred
7. Nine millions four thousand one hundred and twenty-eight.
8. Eighty-seven millions, nine hundred thousand.
9. Nine hundred and seventy-eight millions, three hundred and sixty-five.

When numbers, consisting of many figures, are given to be read, it will be found convenient to divide them into periods of six figures each, calling the first or right hand period that of units; the second that of millions; the third billions; the fourth trillions, &c. as in the following

TABLE.

Hundreds of thous. of Trillions.	Tens of thousands of Trillions.	Thousands of Trillions.	Hundreds of Trillions.	Tens of Trillions.	TRILLIONS.	Hundreds of thous. of Billions.	Tens of thousands of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	BILLIONS.	Hundreds of thous. of Millions.	Tens of thousands of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	MILLIONS.	Hundreds of Thousands	Tens of Thousands.	Thousands.	Hundreds	Tens.	Units.
4	6	5	2	1	9,	6	7	8	3	9	1,	5	4	3	6	7	5,	2	1	3	4	2	8
4th Period or Period of Trilions.						3d Period or Period of Billions.						2d Period or Period of Millions.						1st Period or Period of Units					

The foregoing number is read thus. 465 thousand 219 trillions, 678 thou

sand 391 billions, 543 thousand 675 millions, 213 thousand four hundred and twenty-eight.

Trillions is succeeded by Quatrillions, Quintillions, Sextillions, Septillions, Octillions, Nonillions, Decillions, Undecillions, Duodecillions, &c.

N. B. Billions is substituted for millions of millions; trillions for millions of millions of millions; quatrillions for millions of millions of millions of millions, &c.

Questions.

1. What is Arithmetic? How many principal rules has it for operation?
2. What does numeration teach?
3. What is a single or individual thing called?
4. What are the characters made use of in computation?
5. What are the nine first of these characters called; and what values have they?
6. When standing separately or alone, what value do they express?
7. When placed or located with other figures what value have they?
8. What does any figure in the first place; or place of units express? What, in second place or place of tens? What in the third place?
9. What does the cipher signify or represent?
10. What is the rule for reading numbers?
11. What is the rule for writing numbers?

ADDITION.

1. If you have two apples in one hand, and one in the other, how many have you in both?

2. If you have four pins in one hand, and James puts in three more, how many will you then have?

3. If James has five apples, and George has four, how many have they both together?

4. Thomas gave nine cents for a purse, and had five cents left to put into it; how many cents had he at first?

5. A boy paid twelve cents for paper, and five cents for quills; what was the amount paid for both?

6. If you had twelve cents, and should find six more, how many would you then have?

7. Peter bought a book for ten cents, and sold it again so as to gain six cents; how much did he get for it?

8. A farmer sold seven cows, and had nine left; how many had he at first?

9. A man bought 12 bushels of corn for nine dollars, and 10 bushels of rye for seven dollars, how much did he pay for both?

10. If you have three cents in one hand, two cents in the other hand, and five cents in your pocket, how many do they all make?

11. If you give twelve cents for a knife, eight cents for

an ink stand, and five cents for quills, how many cents do they all come to?

How many are 12, 8, and 5?

SIMPLE ADDITION.

Simple Addition teaches to collect, or put together, several smaller numbers of the same denomination into one sum; as \$8 and \$4 in one sum are \$12.

RULE.

1. Write the numbers, units under units, tens under tens, &c. and draw a line under them.

2. Begin at the right hand, or column of units, and add up every figure in that column, and if the amount does not exceed nine, write it under the column; but if the amount be greater than nine, set down the right hand figure or units, and carry the left hand figure or figures, which are tens, to the next column of tens: proceed in the same manner through every column or row, and set down the whole amount of the last.

PROOF. Begin at the top of the sum and reckon the figures downwards; or, cut off the upper line of figures, and find the amount of the rest, then if the amount and upper line, when added, be equal to the sum total, the work is supposed to be right.

EXAMPLES.

1. What is the whole sum of 312 dollars, 32 dollars, 511 dollars, and 123 dollars?

Operation.

	Hunds.	Tens.	Units.
	3	1	2
		3	2
	5	1	1
	1	2	3
Ans.	9	7	8

We write the numbers one under another, so that units may stand under units, tens under tens, &c. We then add up the column of units, thus, 3 and 1 are 4 and 2 are 6 and 2 are 8, which we set down under the column of units. We next proceed to add up the column of tens. 2 and 1 are 3 and 3 are 6 and 1 are 7, which we write under the tens.

We then add up the column of hundreds in the same manner and find the amount to be 9, which we set down, and the work is done.

2. Add together the following numbers, viz: 597, 236, 53 and 439.

	Hunds.	Tens.	Units.
	5	9	7
	2	3	6
		5	3
	4	3	9
1	3	2	5

We add up the units, thus, 9 and 3 are 12 and 6 are 18 and 7 are 25 units. (=2 tens and 5 units.) We set down the right hand figure 5, units under the units, and carry or add the left hand figure, 2 tens, in with the other tens. Thus, we carry 2 to 3 are 5 and 5 are 10 and 3 are 13 and 9 are 22 tens, setting down the right hand figure 2 under the tens, and carrying the left hand figure 2 hundreds to the hundreds; thus, carry 2 to 4 are 6 and 2 are 8 and 5 are 13, setting down the whole amount

(3)

Tens.	Units.
5	3
1	3
4	2
3	5
8	9

(4)

Hundreds.	Tens.	Units.
1	4	4
2	8	9
8	5	1
2	1	5
8	9	6

(5)

Thousands.	Hundreds.	Tens.	Units.
1	8	6	5
3	0	4	2
7	2	4	4
5	4	2	1
8	9	5	6

(6)

	Hunds. of Thous.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units
	4	4	2	3	6	1
	1	3	5	7	8	9
	8	4	0	3	2	7
	3	1	2	4	1	4
	1	3	2	4	6	5
1	8	6	3	3	5	6

(7)

1	3	2	4	6	5
7	3	1	4	5	3
9	8	4	7	5	6
8	4	3	6	9	8
6	5	4	2	7	3
4	0	3	2	1	2

(8)

6	4	0	7	8	9
9	8	7	4	2	3
1	2	5	6	9	9
4	1	1	3	4	5
5	2	5	1	3	4
8	7	9	0	5	3

(9)

2	1	1	0	0	9
4	0	7	3	8	4
6	7	5	5	6	0
5	9	4	3	0	7
7	8	5	3	2	1
8	0	3	0	6	3

(10)	(11)
2446508736	9706443962
4503260975	853085627
5816953049	49506439
8909767085	9076804 63
9073843098	509208
7657982433	914091

12. Add together the following numbers, 152, 143, 56, 25 and 8. *Ans.* 384.

13. Find the sum total of 536, 213, 118, 95, 25 and 13. *Ans.* One thousand.

14. What is the whole amount of 3482, 783645, 318, 7530 and 9678045 added together? *Ans.* 10473020.

15. Add together the following numbers, 1118, 1469, 784645 and 956684. *Ans.* 1743916.

16. What is the sum total of 1650 dollars, 1975 dollars, 146 dollars, 3125 dollars, 918 dollars and 560 dollars? *Ans.* $8374.

17. Find the sum of 8301, 9461, 12648, 1275 and 8315. *Ans.* Forty thousand.

18. What is the whole sum of 10356, 204560, 126432, 84031, 978402 and 8596218? *Ans.* 9999999.

19. Find the amount of the following sums of money, viz: 78 dollars, 48 dollars, 68 dollars, 58 dollars, 35 dollars, 85 dollars, 94 dollars and 34 dollars. *Ans.* $500.

20. Add 784645, 97632, 548301, 750562 and 318860 into one sum. *Ans.* Two millions five hundred thousand.

21. What is the sum total of the following numbers, viz:
Four hundred and eighty-six,
Five thousand five hundred and eight,
Eighty-four thousand five huudred,
Eight hundred and eleven thousand,
Eight millions and thirty-six?

Answer, 8901530.

22. Write down and add together the following numbers, viz: Nine hundred and twenty-seven,
Two thousand three hundred and forty-eight,
Fifty-two thousand six hundred,

Three hundred and twenty-five thousand,
Nine millions two hundred thousand,
Seventy-nine millions three hundred and nineteen.

Answer, 88581194.

MORE EXAMPLES FOR EXERCISE.

1. A certain farm is divided into five lots; the first lot contains 114 acres, the second 98 acres, the third 125 acres; the fourth 215 acres and the fifth 168 acres; how many acres does the whole farm contain? *Ans.* 720 acres.

2. General Washington was born in the year 1732, and lived to be 67 years of age; in what year did he die? *Ans.* 1799.

3. What number of dollars are in six parcels, each containing 25479 dollars? *Ans.* 152874 dollars.

4. If one quarter of a ship and cargo be worth twelve thousand four hundred and forty-eight dollars, what is the whole ship and cargo worth? *Ans.* 49792 dollars.

5. If one man own one-sixth of a bank, and his part cost 25000 dollars, what is the bank worth at that rate? *Ans.* 150000 dollars.

6. The distance from Portland, in the State of Maine, to Boston is 118 miles; thence to Hartford 100 miles; thence to New-York 123 miles; thence to Philadelphia 90 miles; thence to Baltimore 100 miles; thence to Washington 38 miles; thence to Richmond 123 miles; thence to Raleigh 165 miles; thence to Charleston 256 miles; thence to Savannah 113 miles. How many miles from Portland to Savannah? *Ans.* 1226 miles.

7. A man divided his whole estate equally among his five children, giving each one three thousand six hundred and forty-five dollars; what did his whole estate amount to? *Ans.* 18225 dollars.

8. If from the creation of the world to the flood were 1650 years; from that to the calling of Abraham 427 years; from that to the building of Solomon's Temple 909 years; from that to the foundation of Rome 266 years; from that to the birth of Christ 752 years; how many years from the creation of the world to the birth of Christ? *Ans.* 4004.

9. What is the sum of twenty-five; five hundred and thirty-six; eight thousand eight hundred; fifty-six thousand; nine hundred and eighty-four thousand; eight millions; seventy-nine millions and forty-five; five hundred eighteen millions; nine hundred seventy-six millions eight hundred forty-four thousand five hundred and sixty-eight?

Ans. 1582893974.

Questions.

What does Simple Addition teach? How do you write the numbers to be added? Where do you begin to add? If the amount of any column when added does not exceed 9 what do you do? If it exceeds 9 what would you set down? How do you set down the amount of the last row? How do you prove addition? What is the sign of addition? What is the sign of equality?

FEDERAL MONEY.

Federal Money is the coin or currency of the United States. This is the most simple and easy of all currencies. It increases in a tenfold proportion like whole numbers.

Thus	10 mills (*m.*) make		1 cent, marked	c.
	10 cents make		1 dime,	d.
	10 dimes, or 100 cents,	make	1 dollar,	$.
	10 dollars make		1 Eagle,*	E.

Hence it follows that any sum in federal money may be performed as in whole numbers

The dollar is the money unit; and to distinguish dollars from the smaller denominations, a point, or comma (,) called a separatrix, is placed between them and the other inferior denominations; consequently, the first figure at the left hand of the separatrix is dollars and the rest are eagles. And the first figure at the right hand is dimes, the second cents, and the third mills.

But as the denominations of this Money increase in a

* The Eagle is a gold coin. Dollars and dimes are silver coins, and cents are copper coins. These are real coins. There are also half Eagles, half dollars, half dimes, and half cents, real coins.

There is no coin so small as the mill, that being only imaginary. Mill is contracted from Mille, the Latin for thousands; cent from centum, the Latin for hundreds; and dime from disme, the French for tenth.

tenfold proportion like whole numbers, it is evident that all the figures at the left of the separatrix, may be called dollars, and those at the right, cents and mills. That is, the first two at the right, cents, and the third mills; and this is the usual way of expressing it in commercial business.

ADDITION OF FEDERAL MONEY.

RULE.

1. Write the numbers, dollars under dollars, dimes under dimes, cents under cents, &c. and add as in whole numbers.

2. Then place the separatrix in the sum total, exactly under the separating points above.

EXAMPLES.

$. d. c. m.	$. d. c. m.	$. d. c. m.
2 4 1, 5 6 8	1 3 5 7, 5 7 9	9 8 7, 2 5 6
1 1 3, 4 3 6	5 4 7 8, 6 4 8	3 4 6, 4 1 5
5 6 8, 1 2 3	3 1 4 2, 2 3 1	6 0 5, 7 5 0
6 4 2, 3 4 2	8 7 9 4, 9 8 7	1 2 5, 3 7 4
4 1 0, 6 7 1	2 3 5 3, 7 3 6	4 1 2, 2 5 0
3 6 5, 3 5 4	6 7 3 4, 3 5 3	8 7 9, 4 9 5
2 3 4 1, 4 9 4		

Note. In writing down dollars and cents, when the cents are less than 10, put a cipher on the left, or in the tens' place; thus, 4 dols. and 6 cts. are written $4,06, &c.

	$. cts. m.
4. What is the sum total of 112 dols. 9 cts.,	1 1 2, 0 9 0
35 dollars 91 cents and 3 mills,	3 5, 9 1 3
48 dollars 75 cents and 4 mills,	4 8, 7 5 4
And 105 dollars 7 cents and 5 mills?	1 0 5, 0 7 5
Ans.	3 0 1, 8 3 2

equal to 301 dollars 83 cents 2 mills, which may also be read either of the following ways, viz: 30 eagles 1 dollar 8 dimes 3 cents 2 mills, or 301 dollars 8 dimes 3 cents 2 mills, or 30183 cents 2 mills, or 301832 mills.

5. What is the sum of 57 dollars 20 cents, 6 dollars 2 cents, 81 dollars 16 cents, 70 dollars 19 cents, 62 dollars 9 cents, and 55 dollars 3 cents? Ans. $331,69.

6. Find the sum of 20 cents, 40 cents, 30 cents, 50 cents, and 60 cents. *Ans.* 2 dollars.

7. Add together 311 dollars, 48 dollars 9 cents, 225 dollars 13 cents, 44 dollars 3 mills, and 13 dollars 84 cents and 5 mills. *Ans.* $642,068.

8. A merchant is indebted to A $145,91, to B $345,11, to C $918,85, to D $1144,75, and to E $1690,84; what is the whole amount of his debts? *Ans.* $4245,46.

9. If one third of a vessel be worth 10145 dollars 95 cents, what is the whole worth at that rate? *Ans.* $30437,85.

10. What is the sum of 325 dollars 41 cents, 695 dollars 5 cents, 84 dollars, 411 dollars 18 cents, 725 dollars 58 cents 5 mills, and 123 dollars 70 cents 5 mills?

Ans. $2364,93.

11. A received 411 dollars, 88 cents and B received just three times as much money as A received; how much money did B receive? *Ans.* 1235,64.

12. B paid me $98,56, C paid me just four times as much as B, and D paid me just as much as B and C both; can you tell how much money D paid me? *Ans.* $492,80.

13. If one eighth of a bank be worth thirty-two thousand five hundred and twelve dollars forty-five cents, what are four eighths, or one half of it, worth at the same rate?

Ans. $130049,80.

Questions.

What is Federal Money? How does it increase? How many mills make a cent? How many cents make a dime? How many dimes make a dollar? How many dollars make an eagle? How many cents make a dollar?

Which is the money unit? How are dollars distinguished from the smaller denominations? What is the first figure on the left hand of the separatrix? What are the rest? What is the first figure on the right hand of the separatrix? The second? The third? What may all the figures on the left of the separatrix be called? What will be the first two on the right? What the third? How do you write the numbers? How do you add? Where do you place the separatrix in the sum total?

SUBTRACTION.

1. George had 10 apples and gave 6 of them to Henry, how many had he left? *Ans.* 4. Why? Because 6 taken from 10 leave 4, or 4 added to 6 make 10.

2. A boy having 9 cents paid 5 for apples, and lost the remainder, how many did he lose?

3. James had 12 apples and John 7, how many more had James than John?

4. Peter has 9 cents and wishes to buy a book worth 17 cents, how many cents does he lack?

5. A merchant bought a box of goods for 10 dollars and sold it for 17 dollars, how much did he gain?

6. James bought a book and pencil for 14 cents; he gave 9 cents for the book: what did the pencil cost?

7. Stephen sold a knife for 16 cents, which was 7 cents more than he gave for it; how much did he give for it?

8. A boy being asked how old he was, said that he was 7 years younger than his brother whose age was 16 years, how old was he?

9. If you had 17 cents and should lose 8 of them, how many would you have left?

10. A man owed 16 dollars and paid 9 dollars, how much remains due?

11. A lady took with her 15 dollars to buy goods and returned with 6 dollars, how much did she lay out?

12. A farmer bought a cow and calf for 19 dollars, and afterwards sold the calf for 5 dollars; what did the cow stand him in?

13. A market woman had 30 oranges and sold 15 of them, how many had she left?

14. A certain school contains 40 scholars 15 of whom are girls; how many boys are there?

The scholar may now proceed to the following Rule and Illustration.

SIMPLE SUBTRACTION.

Simple Subtraction teaches to take a less number from a greater; or to find the difference between any two numbers of the same name or denomination; as 5 dollars subtracted from 8 dollars, the remainder is 3 dollars.

RULE.

1. Write the less number under the greater, so that units may stand under units, tens under tens, &c., and draw a line under them.

2. Begin with the units and subtract each figure in the lower line from the figure over it; and set down the remainder.

3. When any figure in the lower line is greater than the figure above it, add 10 to the upper figure and from that amount subtract the lower figure and set down the remainder; observing when you so increase the upper number by 10 always to add 1 to the next lower figure before subtracting, and thus proceed until the whole is finished.

Proof. Add the remainder to the less number, and if the sum be equal to the greater number, the work is right.

Note. The greater number is sometimes called the minuend, and the smaller the subtrahend.

EXAMPLES.

1. What is the difference between 1856 and 642?

Operation.
Minuend 1856
Subtrahend 642
——
Difference 1214

We place the less number under the greater, units under units, tens under tens, &c. then take 2 units from 6 units leave 4 units; and 4 tens from 5 tens leave 1 ten; and 6 hundreds from 8 hundreds leave 2 hundreds, and nothing from 1 thousand leaves 1 thousand; and the answer is 1214

2. Subtract 528 from 873 and how many remains?

Greater num. 873
Less number 528
——
Remainder 345
——
Proof 873

It is evident we cannot take 8 units from 3 units, but will suppose the upper number 3 to be increased by 10, that is, we will take 1 ten off of the 7 tens, and add it to the 3 units, making 13, and then subtract 8 from 13, leaves 5. (We have now borrowed 1 ten from the 7 tens, and in reality it is only 6 tens from which we are to subtract the 2 tens in the lower number.) We will now carry or add 1 to the next lower figure 2 tens, makes 3 tens; and 3 from 7 leaves 4. Then subtract 5 (hundreds) from 8 (hundreds) leaves 3 (hundreds) and the work is done.

Note. It will be seen that carrying 1 to the lower figure 2 tens and subtracting it from the 7 tens produces the

same effect; and is more convenient in practice than it would be to consider the upper figure (7 tens) from which we borrowed 1 ten; to be only 6 tens, and subtracting the lower figure 2 tens from it; thus, 2 from 6 leaves 4, the same as above. Hence is derived the Rule. "When the lower figure is greater than the figure above it add 10 to the upper figure," &c.

Proof. The remainder and less number added together are equal to the greater number, therefore the work is right.

	3		4		5
Greater number	856	Minuend	4778	From	47784
Less number	723	Subtrahend	987	Take	4775
Remainder	133	Difference	3791	Remains	43009

	6	7	8	9
From	72157	807647	5768475	14678933
Take	13148	174348	857643	7459786
Remain.				

	10	11	12	13
From	516745	1000000	100000	6662666
Take	200000	909000	8	5777778
Rem.				

14. From 784656 take 645485 *Ans.* 139171.
15. From 913417 take 246751 *Ans.* 666666.
16. From 5456841 take 1209825 *Ans.* 4247016.
17. From 3356948 take 2468009 *Ans.* 888939.
18. From 479660 take 146327 *Ans.* 333333.
19. From 1000000 take 127653 *Ans.* 872347.
20. From 1225683 take 225684 *Ans.* 999999
21. From three millions take one million five hundred thousand. *Ans.* 1500000.
22. If the minuend be 95657 and the subtrahend 84465 what is the difference or remainder? *Ans.* 11192.
23. If the minuend be 1002243 and the subtrahend 667788 what is the remainder? *Ans.* 334455.

24. From fifty-five thousand and thirty-six take five thousand and twenty-one. *Ans.* 50015.

25. Can you tell me the difference between one million and nine hundred and ninety-nine thousand?

Ans. One thousand.

EXERCISES IN ADDITION AND SUBTRACTION.

1. A man bought a ship for 25311 dollars, and sold her for 34728 dollars, what did he gain by the purchase? *Ans.* 9417 dollars.

2. How much is 1200 greater than 365 and 721 added together? *Ans.* 114.

3. If I have 3541 dollars, and pay A 1200 dollars and B 975 dollars out of it, how many dollars shall I have left? *Ans.* 1366.

4. What number added to 9709 will make 10000?

Ans. 291.

5. General Washington was born in the year 1732 and died in the year 1799, how old was he when he died?

Ans. 67.

6. The whole population of the six New-England States in the year 1800 was 1,232,454; in 1830 it was 1,954,220; what was the increase in 30 years? *Ans.* 721766.

7. What other numbers with these four, viz: 21, 32, 16, and 12, will make 100? *Ans.* 19.

8. A gentleman's whole estate amounted to eleven thousand five hundred dollars; and he gave his two sons Charles and Edward each four thousand five hundred dollars, and the remainder to his daughter Eliza; what was Eliza's share? *Ans.* $2500.

9. From Hartford to New York is 123 miles, thence to Philadelphia 90 miles, thence to Washington 138 miles: now if a man travel 6 days from Hartford towards Washington, at the rate of 47 miles a day, how far will he then be from Washington? *Ans.* 69 miles.

10. A wine merchant bought 843 pipes of wine for 97530 dollars, and sold 684 pipes thereof for 89631 dollars; how many pipes had he left, and what do they stand him in?

Ans. 159 pipes left, and they stand him in $7899

SUBTRACTION OF FEDERAL MONEY.

RULE.

Write the less number below the greater, dollars under dollars, dimes under dimes, cents under cents, &c. and subtract as in Simple Subtraction.

EXAMPLES.

1. From 54 dollars 7 cents 4 mills, take 48 dollars 68 cents 5 mills. *Ans.* $5,3d. 8cts. 9m. or 5 dollars 38 cents 9 mills.

Operation.	$. cts. m
Greater number	54,07,4
Less number	48,68,5
Remainder	5,38,9

	$ d.c.	$ d.c.m.	$ cts.
From	54,7,8	368,4,5,6	980,00
Subtract	35,8,7	257,3,2,7	640,01
Rem.	18,9,1		

	$ c.m.	$ c.	$ c. m.
From	4568,37,5	5307,73	1607,01,9
Take	695,78,4	431,29	311,50,4

8. From 135 dollars take 8 dollars 8 cents. *Ans.* $126,92.

9. From 154 dollars 1 cent take 51 dollars 10 cents. *Ans.* $102,91.

10. From 348 dollars 90 cents take 158 dollars 99 cents. *Ans.* $189,91.

11. From 100 dollars take 99 cents. *Ans.* $99,01.

12. From 99 dollars 9 cents take 9 dollars 90 cents. *Ans.* $89,19.

13. From $50,10 take $19,01. *Ans.* $31,9.

14. From 783 dollars 48 cents subtract 625 dollars. *Ans.* $158,48.

15. From $51, take 42 cents. *Ans.* $50,58.

16. From 25 dollars subtract 25 cents. *Ans.* $24,75.

17. From 40 dollars take 40 cents 1 mill. *Ans.* $39,59,9.

18. From $1584,01 subtract $45,91. *Ans.* $1538,10.

19. Subtract 1 mill from $100. *Ans.* $99,99,9.

20. From 30 dollars 75 cents take 74 cents 1 mill. *Ans.* $30,00,9

21. Borrowed 150 dollars, and paid 45 dollars 91 cents; how much remains unpaid? *Ans.* $104,09.

22. Charles received $135,41 and Henry received just three times as much lacking 94 cents; how much money did Henry receive? *Ans.* $405,29.

23. A man received $1500, and out of which he paid to A $511,45, to B $251,15, to C $180,39, and to D $205,98; how much had he left after paying his landlord's bill of board, amounting to 35 dollars 67 cents? *Ans.* $315,36.

Questions.

1. What does Simple Subtraction teach?
2. How do you write the numbers?
3. Where do you begin to subtract?
4. When any figure in the lower line is greater than the figure above it, what would you do? When you add 10 to the upper figure, what would you do with the next lower figure before you subtract it?
5. What is this called? Ans. borrowing 10. How do you prove Subtraction?
6. What is the greater number sometimes called?
7. What is the smaller number called?
8. What is the Rule for Subtraction of Federal Money?
9. What is the sign of Subtraction?

MULTIPLICATION.

1. James has 5 cents, and Edward has three times as many; How many has Edward? 3 times 5 are how many?

2. If 1 orange cost 5 cents, how many cents will 6 oranges cost? 6 times 5 are how many? 5 times 6 are how many?

3. One bushel contains 4 pecks; how many pecks are there in nine bushels? 9 times 4 = how many? 4 times 9 are how many?

4. At 3 dollars a yard what will 8 yards of cloth come to? 8 times 3 are how many? 3 times 8 are how many?

5. At 7 dollars a barrel what will 8 barrels of flour come to? 8 times 7 are how many? 7 times 8 are how many?

6. What will 4 cows come to, at 12 dollars a head? *Ans.* $48.

7. Three persons purchased a garden, each paid 23 dollars; how much did the garden cost? *Ans.* 69 dollars because 3 times 3 units are 9 units, and three times 2 tens are 6 tens, and 6 tens and 9 units make 69, the answer.

8. How much will 5 bushels of apples come to at 22 cents per bushel? *Ans*. 110 cents.

Solution—22 is 2 tens and 2 units, and 5 times 2 units are ten units or 10; and 5 times 2 tens are 10 tens, and 10 tens and 1 ten make 11 tens=110, the answer.

9. There is an orchard containing 5 rows of trees, in each row are 23 trees; how many trees does the orchard contain?

In the first row	23	trees
second	23	
third	23	
fourth	23	
fifth	23	
In the whole	115	trees

Multiplicand	23
Multiplier	5
Product	115

By this we see that we may obtain the whole number of trees by setting down 23, 5 times and adding it up, making 115 trees in the whole.

But we need only set down the number of trees contained in 1 row, and as 5 rows will contain 5 times as many, we may place the 5 underneath for a multiplier; we know that 5 times 3 units are equal to 15 units or 1 ten and five units; we set down the 5 units and reserve the 1 ten; then 5 times 2 tens are 10 tens, and 1 ten which we reserved makes 11 (tens) which set down, and the answer is 115, as before. Hence we find that Multiplication is a short way of performing Addition.

SIMPLE MULTIPLICATION

Teaches to repeat the greater of two simple numbers as many times as there are units in the less or multiplying number, or it is a compendious method of performing many additions.

The number to be multiplied is called the multiplicand.

The number you multiply by, is called the multiplier.

The number found by the operation, is called the product.

Both multiplier and multiplicand are called factors.

Note. It will not make any difference in the result of the multiplication, whether we make the greater number the

multiplicand, and the less number the multiplier, or the less number the multiplicand and the greater number the multiplier; for the product of any two factors will be the same in either case: (thus 5 taken 3 times is the same as 3 taken 5 times.) But in practice it is moreconvenient to make the greater number the multiplicand and the less, the multiplier.

CASE I.

When the multiplier is not greater than 12.

RULE.

Place the multiplier under the multiplicand, and multiply each figure in the multiplicand by the multiplier, setting down and carrying as in Simple Addition.

EXAMPLES.

1. There are 365 days in one year; how many days are there in three years?

Illustration—It is evident that if one year contains 365 days, three years will contain 3 times 365 days.

Operation.

365 multiplicand	We write down the multiplicand and place the multiplier under it, making
3 multiplier	the greater number the multiplicand;
1095 Product	the less the multiplier.

We then say 3 times 5 are 15 setting down the 5 (units) and carrying 1 (ten,) as in Simple Addition; then 3 times 6 are 18 and 1 to carry makes 19, setting down 9 and carrying 1 to the next, we say 3 times 3 are 9 and 1 makes 10, setting down the whole product. Hence we find that 3 times 365 is equal to 1095. We might have obtained this same answer by setting down 365 the multiplicand 3 times and adding it up.

	2	3	4	5	6
Multiplicand	57436	5432	2345	9054	152634
Multiplier	2	3	4	5	6
Product	114872				

	7	8	9	10	11
Multiplicand	17048	81034	5419	41261	4230
Multiplier	7	8	9	11	12
product					

11. If a man travel 48 miles a day, how many miles will he travel in 9 days? *Ans.* 432 miles.

12. What is the product of 956 multiplied by 12? *Ans.* 11472.

13. What will 325 barrels of flour come to at 7 dollars a barrel? *Ans.* $2275.

14. 320 rods make a mile; how many rods are there in 8 miles? *Ans.* 2560.

CASE II.

When the Multiplier consists of several figures.

RULE.

1. Write the multiplier under the multiplicand, placing units under units, tens under tens, &c.

2. Then multiply by each significant figure in the multiplier, separately, and place the first figure in each product, directly under its multiplier.

3. Add together the several products in the same order as they stand, and you will have the total product.

*Proof.**—Multiply the multiplier by the multiplicand.

EXAMPLES.

1. There are 365 days in one year; how many days are there in 25 years?

	Operation.
Days in 1 year	365
Years	25
	1825
	730
Days in 25 yrs.	9125

Illustration.—We first multiply each figure in the multiplicand by the 5 (units) in the multiplier, which gives the product of 5 times 365 or the number of days in 5 years.

We then multiply each figure in the multiplicand by the 2 tens in the multiplier: placing the first figure in

* Another method of Proof is as follows, viz.: cast out the 9's in both factors, and set down the remainder at the right hand; then if the excess of 9's in the two remainders when multiplied together be equal to the excess of 9's in the total product, the work is supposed to be right.

Thus,	365 = 5 excess of 9's.	
	29 = 2 excess of 9's.	
	3285	10 excess = 1
	730	
	10585	excess = 1

the product directly under its multiplier; and as the value of any number is increased ten times by being moved one place to the left, it is evident that the product found by multiplying by 2 in the ten's place is equal to 20 times 365 or the number of days in 20 years; and the product of 5 years and 20 years added together are equal to 9125 days or the number of days in 25 years.

	2	3	4
Multiplicand	856	1234	1836
Multiplier	18	25	32
	6848	6170	3672
	856	2468	5508
Product	15408	30850	58752

	5	6	7
Multiply	36494	45085	129186
by	74	91	98
Product	2700556	4102735	12660228

```
 37687
   205
188435
75374
7725835
```

When there are ciphers between the significant figures in the multiplier as in this example; omit the ciphers, and multiply by the significant figures only; placing the first figure in each product under its multiplier, as before.

8. Multiply 3672 by 508 Product 1865376.
9. Multiply 543764 by 239 " 129959596.
10. Multiply 181281 by 763 " 138317403.
11. What number is equal to 3027 times 82164973? *Ans.* 248713373271.
12. Multiply 27501976 by 271. *Ans.* 7453035496.
13. Multiply 8496427 by 874359. *Ans.* 7428927415293.
14. Multiply 95644796 by 8000009. *Ans.* 765159228803164.
15. Multiply 562916859 by 49007. *Ans.* 27586866509013.

CASE III.

When ciphers stand at the right hand of either or both of the factors, neglect them, and place the significant figures under one another and multiply by them only; then place as many ciphers on the right hand of the product, as were neglected in both factors.

EXAMPLES.

Multiply	293	568000	7554000
by	700	84	3400
Product	205100	47712000	25683600000

4. Multiply 4568 by 900. *Ans.* 4111200.
5. How many are 800 times 2567? *Ans.* 2053600.
6. Multiply 29526000 by 4030. *Ans.* 118989780000.
7. Multiply 596780000 by 98200. *Ans.* 58603796000000.
8. 930137000×9500=8836301500000.
9. 819600000×591800000=485039280000000000.

When the Multiplier is 10, 100, 1000, &c.

Since numbers increase in a tenfold proportion from the right hand to the left, it is evident that if we place one cipher on the right hand of any figure it increases the value of that figure ten times by removing it to the place of tens; and if we place two ciphers on the right of any figure it increases its value a hundred fold by removing it to the place of hundreds. Hence the following

RULE.

Place the ciphers of the multiplier on the right of the multiplicand and it makes the product required.

EXAMPLES.

1. Multiply 56	by 10,	the product is	560
2. Multiply 548	by 100		54800
3. Multiply 85	by 1000		85000
4. Multiply 584	by 10000		5840000
5. Multiply 9640	by 100000		964000000

CASE IV.

When the Multiplier is a Composite Number.

A composite number is a number that can be produced by multiplying two or more numbers together: thus, 24 is a composite number, and its component parts or factors may be 4 and 6, which multiplied together are equal to 24; or they may be $2\times3\times4=24$, or $3\times8=24$, &c.

RULE.

1. Find two or more numbers which multiplied together will, exactly make the multiplier.
2. Then multiply by one of those factors and that product by another, &c., and the last product will be the Answer.

EXAMPLES.

1. What will 35 acres of land come to at 56 dollars an acre?

56=price of 1 acre.
7=one of the component parts or factors of 35.
392=price of 7 acres.
5=the other factor.
Ans. 1960=price of 5 times 7 or 35 acres.

2. Multiply 576421 by 36,	*Ans.* 20751156.
3. Multiply 764131 by 48,	*Ans.* 36678288,
4. Multiply 844792 by 64,	*Ans.* 54066688,
5. Multiply 91738 by 81,	*Ans.* 7430778,
6. Multiply 43564 by 108,	*Ans.* 4704912.
7. Multiply 814352 by 144,	*Ans.* 117266688.
8. Multiply 218541 by 112,	*Ans.* 24476592,

NOTE. 8, 7, and 2 are factors of 112.

EXAMPLES FOR PRACTICE.

1. If 35 men receive each 137 dollars, how many dollars will they all receive? *Ans.* 4795.

2. In a certain orchard there are 125 rows of trees, each row containing 96 trees; how many trees does the orchard contain? *Ans.* 12000.

3. How much will 145 tons of hay come to at 15 dollars a ton? *Ans.* $2175.

4. It takes 320 rods to make 1 mile; how many rods are there in 596 miles? *Ans.* 190720

5. If a ship sail 192 miles a day, how many miles will she sail in 56 days? *Ans.* 10752 miles.

6. A merchant bought 46 bales of cloth, each bale containing 35 pieces, and each piece 28 yards; how many yards did he buy in all? *Ans.* 45080.

7. Multiply 35860205 by 365, *Ans.* 13088974825.

8. Multiply 2703682 by 8409, *Ans.* 22735261938.

9. Multiply 15569800 by 8300, *Ans.* 129229340000.

Questions.

1. What does Simple Multiplication teach? What does it perform the work of?

2. What is the number to be multiplied called? What is the number you multiply by, called?

3. What is the number found by the operation, called? What are the multiplier and multiplicand called?

4. When the multiplier is not greater than 12, how do you multiply?

5. When the multiplier consists of several figures, how do you write it down? In multiplying, where do you place the first figure in the product?

6. What do you do with the several products?

7. How do you prove multiplication?

8. When there are ciphers on the right hand of the factors, how do you multiply?

9. How do you multiply by 10, 100, 1000, &c.?

10. What is a composite number?

11. When the multiplier is a composite number, how do you multiply?

12. What is the sign of multiplication? Let me see you write it down.

DIVISION.

1. If you divide 15 apples equally among 3 boys, how many will each have? How many times 3 in 15?

2. If 15 apples be divided equally among 5 boys, how many will each have? 5 in 15, how many times?

3. James has 16 cents to buy pencils with; how many can be buy at 4 cents apiece? How many times is 4 cents contained in 16 cents?

4. How many oranges, at 6 cents apiece, can you buy for 36 cents? 6 in 36 how many times?

5. A man bought 9 lemons for 81 cents; how much did he give apiece? 9 in 81 how many times?

6. How many barrels of cider, at 3 dollars a barrel, can be bought for 27 dollars?

7. In an orchard there are 48 trees, standing in 8 rows; how many trees are there in a row? How many times is 8 contained in 48?

8. How many barrels of flour can you buy for 84 dollars, at 7 dollars a barrel?

9. If 9 yards of cloth cost 27 dollars, what is 1 yard worth?

10. A man worked 8 days for 40 shillings; how much was that a day?

11. A man divided 72 cents equally among 18 poor boys; how many cents did he give to each?—He divided them as follows, viz:

	72
1. He gave 1 cent to each, making	18
	—
He then had	54 left.
2. He gave another cent to each =	18
	—
He then had	36 left.
3. He gave another cent to each =	18
	—
He then had	18 left.
4. He gave another cent to each =	18
	—
	0 left.

We find that 18 can be subtracted from 72, 4 times; therefore 18 is contained in 72, 4 times.

By this we see that the operation may be performed by subtraction; but when the sum is large, and would require a great many subtractions, the operation is more easily performed by the Rule called Division.

SIMPLE DIVISION

Teaches to find how many times one simple or whole number is contained in another, and also what remains Hence it performs the work of several subtractions in a concise manner.

There are four principal parts in division, viz:

1. The Dividend, or number given to be divided.

2. The Divisor, or number given to divide by.

3. The Quotient, or answer to the question, which shows how many times the divisor is contained in the dividend.

4. The Remainder, which is always less than the divisor, and of the same name of the dividend.

SHORT DIVISION

Is when the divisor does not exceed 12.

RULE.

1. Find how many times the divisor is contained in the first left hand figure, or figures of the dividend; place the figure expressing the number of times under the dividend, and carry the remainder, as so many tens to the next figure and divide the sum it makes as before; and if the divisor is not contained in this sum, place a cipher under it, and carry the whole as so many tens to the next figure: thus proceed till you have divided all the figures of the dividend.

Proof.—Multiply the divisor and quotient together, and add the remainder (if there be any) to the product; and if the work be right the sum will equal the dividend.

EXAMPLES.

1. How many yards of cloth, at 3 dollars a yard, can be bought for 654 dollars?

Operation.

	Dividend.
Divisor 3)	6 5 4
Quotient	2 1 8 yds. Ans.

We must find how many times 3 dollars is contained in 654 dollars; that is, we must divide 654 by 3. We find that the divisor 3 is contained in 6, the first figure of the dividend, 2 times. Then 3 is contained in 5, the next figure of the dividend, 1 time and 2 remain. This remainder we carry to the next figure as so many tens; thus, 2 tens =20, which added to 4, the next figure, makes 24, and 3 is contained in 24, 8 times.

In this example, observe, that the 6 which we first divide, means 6 hundred, and the 2 which we place under it means 2 hundred, showing that 3 is contained in 600, 200 times, and the 5 means 5 tens, and the 1 which we place under it means 1 ten, &c.

Proof.

Quotient	2 1 8
Divisor	× 3
Dividend	6 5 4

If 3 contained in 654, 218 times, then it is evident that 218 times 3, or which is the same thing, 3 times 218, will just make 654; and by multiplying them together we find the product to be 654, = the dividend; therefore right.

8. A father left his 6 children 2436 dollars to be equally divided among them; how many dollars had each?

Operation.

	Dividend.
Divisor 6)	2 4 3 6
Quotient =	4 0 6 Ans.

Here 6, the divisor, is not contained in 2, the first figure of the dividend; therefore we join the 2 (thousands) to the 4 (hundreds) making 24 (hundreds;) and 6 is contained in 24 (hundreds) 4 (hundred) times. Then 6 is not contained in 3, the tens of the dividend, therefore we put a cipher under, (that is, in the quotient,) and join the 3 (tens) to the 6 (units,) making 36; and 6 is contained in 36, 6 times. *Ans.* Each had $406

Divisor 2)114351	4)85412	5)94307
Quotient=57175—1 Rem.	21353	
6)142207	7)157215	9)281359

9. How many times is 6 contained in 7326? *Ans.* 1221.
10. How many times is 5 contained in 4565? *Ans.* 913.
11. How many times is 7 contained in 84637? *Ans.* 12091.
12. Divide the number 9784 into 8 equal parts. Quot. 1223.
13. Divide 366024 by 12. Quotient 30502.
14. If 7 dollars will buy 1 barrel of flour, how many barrels of flour may be bought for $3822? *Ans.* 546.
15. A market man received 2943 cents for melons that he sold at 9 cents apiece; how many did he sell? *Ans.* 327.
16. How many times is 7 contained in 6680, and how many over? *Ans.* 954 times, and 2 over.
17. A merchant has $5122 to purchase flour with; how many barrels can he buy at 8 dollars a barrel, and how many dollars will he have left? *Ans.* 640 barrels and 2 dollars left.
18. A prize of 3825 dollars was divided equally among 4 men; how much was each man's part?

Note. We divide the 3825 dollars among the 4 men, and find that each must have 956 dollars, and there is $1 left, which we must divide. Now if we divide 1 dollar into 4 equal parts, each part will be $\frac{1}{4}$ of a dollar.

Ans. Each man must have 956$\frac{1}{4}$ dollars

Had this remainder been 3 dollars, it is evident that each man would have had 3 times $\frac{1}{4}$ of a dollar, that is $\frac{3}{4}$ of a dollar, more ; and in all cases where there is a remainder, we may obtain the true quotient by placing the divisor under the remainder, with a line between, as above.

Thus $\frac{\text{Remainder } 1}{\text{Divisor } \quad 4}$ shows that the divisor 4 is contained in the remainder one fourth of a time.

19. How many cwt. of rice, at 4 dollars a cwt., may be bought for 947 dollars? *Ans.* $236\frac{3}{4}$ cwt

20. How many cwt. of sugar, at 9 dollars per cwt., can be bought for 2944 dollars? *Ans.* $327\frac{1}{9}$.

21. How many barrels of pork, at 11 dollars a barrel, can be bought for 2478 dollars? *Ans.* $225\frac{3}{11}$ barrels.

LONG DIVISION.

When the divisor exceeds 12, we cannot conveniently perform the operation in the mind ; we therefore set the quotient figures on the right hand of the dividend, and write down the whole computation at full length, and this is called long division.

RULE.

I. Find how many times the divisor is contained in the least number of the left hand figures of the dividend, that will contain it once, or more : place the figure expressing the number of times, to the right hand of the dividend for the first quotient figure.

II. Multiply the divisor by this quotient figure, and place the product under that part of the dividend used, and subtract it therefrom.

III. Bring down the next figure of the dividend to the right hand of the remainder, and divide this number as before. Thus proceed till you have brought down, and divided, all the figures of the dividend.

Note. 1. If the product of the divisor by any quotient figure, be greater than that part of the dividend used, it shows that the quotient figure is too large, and must be diminished.—If the remainder at any time be equal to or

greater than, the divisor, the quotient figure is too small, and must be increased.

2. When you have brought down a figure to the right hand of the remainder, if the number made up be less than the divisor, you must place a cipher in the quotient, and bring down the next figure of the dividend.

Proof.—The method of proof is the same as in Short Division.

EXAMPLES.

1. How many times is 13 contained in 2785?

Operation.

```
Divisor. Dividend. Quotient.
 13)2785(214
    26
    --
     18
     13
     --
      55
      52
      --
Remainder 3

Proof.
Quotient   214
Divisor    ×13
           ---
           642
          214
          ----
          2782
Remainder   +3
          ----
          2785=Divid.
```

We find that the divisor (13) is contained in the two first figures of the dividend, 2 times. We place the figure (2) at the right hand of the dividend for the first quotient figure, and multiply the divisor (13) by it, and place the product (26) under the two first figures of the dividend, and subtract it therefrom, and 1 remains. To the right hand of this remainder bring down (8) the next figure of the dividend, which makes 18. We find that 13 is contained in 18, 1 time; placing 1 in the quotient, and 13 under the 18, subtract it therefrom, and 5 remain, to the right hand of which, we bring down (5) the next figure of the dividend, which makes 55. We then find that 13 is contained in 55, 4 times. Placing 4 in the quotient, multiply the divisor (13) by it, and place the product (52) under the 55, subtract it therefrom, and the remainder is 3. Thus we have brought down and divided all the figures of the dividend. Hence we find that 13 is contained in 2785, 214 times, and 3 remains.

2. Divide $2448 equally among 16 men.

Divis. Divid. Quot.
```
16)2448(153
   16
   --
    84
    80
    --
     48
     48
     --
Rem.  0
```

3. How many times is 21 contained in 2859?

Divis. Divid. Quot.
```
21)2859(136 3/21
   21
   --
    75
    63
    --
    129
    126
    ---
Rem.  3
Divisor 21
```

4. Divide 3625 by 18.

Divis. Divid. Quot.
```
18)3625(201
   36
   --
     25
     18
     --
Rem.  7
```

5. How many times is 24 contained in 8760?

Divis. Divid. Quot.
```
24)8760(365 Ans
   72
   --
   156
   144
   ---
    120
    120
    ---
      0
```

6. How many times is 13 contained in 29877? *Ans.* $2298\frac{3}{13}$ times.

7. How many times can you have 16 in 5520? *Ans.* 345 times.

8. If a man travel 630 miles in 15 days, how many miles is that a day? *Ans.* 42 miles.

9. What is the quotient of 55081 divided by 17? *Ans.* $3240\frac{1}{17}$.

10. 25 workmen received $3150, to be equally divided among them; how many dollars did each one receive? *Ans.* $126.

11. If the divisor be 29, and the dividend 72740, what is the quotient? *Ans.* $2508\frac{8}{29}$.

12. How often does 106215 contain 365? *Ans.* 291.

13. Divide 6832 by 16. *Ans.* 427.

14. Divide 28656 by 36. *Ans.* 796.

15. Divide 37088 by 61. *Ans.* 608.

16. Divide 410642 by 79. *Ans.* 5198.

17. Divide 1853219 by 91. *Ans.* $20365\frac{4}{91}$.
18. Divide 9876048 by 821. *Ans.* $30766\frac{162}{321}$.
19. Divide 5740310 by 695. *Ans.* $8259\frac{305}{695}$.
20. Divide 4637064283 by 57606. *Ans.* $80496\frac{11707}{57606}$.

MORE EXAMPLES FOR EXERCISE.

1. Divide 796976499 by 49654. Remainder 29799.
2. Divide 3258675689 by 67435. Remainder 14184.
3. Divide 91876375 by 6493. Remainder 425.
4. Divide 4673854 by 73028. Remainder 62.

CONTRACTIONS IN DIVISION.

CASE I.

When there are ciphers at the right hand of the divisor.

RULE.

Cut off the ciphers in the divisor, and the same number of figures from the right hand of the dividend; then divide the remaining ones as usual, and to the right hand of the remainder, (if any,) place the figures which were cut off from the dividend, and you will have the true remainder.

EXAMPLES.

1. Divide 7380964 by 23100.

```
Divis.   Divid.    Quot.
231'00)73809'64(319 12064/23100
        693
        ---
         450
         231
         ----
         2199
         2079
         -----
          12064
```

2. Divide 8978485 by 8000

```
Divis.   Divid.
8'000)8978'485
Quot.  1122-2485 true rem.
```

3. Divide 741625 by 900.

```
9'00)7416'25
Quot. 824—25 Rem.
```

4. Divide 75694 by 2500. *Ans.* $30\frac{694}{2500}$.

5. Divide 1255000 dollars equally among 5000 men, and how many dollars will each one have? *Ans.* 251.

6. Divide 49184145 by 3600. *Ans.* $13662\frac{945}{3600}$.

7. How many times is 49000 contained in 421400000? *Ans.* 8600.

8. Divide 3259450000 by 60000. *Ans.* $54324\frac{10000}{60000}$.

9. How many times does 11659112 contain 89000? *Ans.* $131\frac{112}{89000}$.

10. Divide 436250000 by 125000. *Ans.* 3490.

MORE EXAMPLES.

1. Divide 149596478 by 120000. Rem. 76478.
2. Divide 654347230 by 901000. Rem. 221230.
3. Divide 457878695 by 9736000. Rem. 286695.

CASE II.

When the divisor is a composite number; that is, when it is the product of any two numbers in the Table.

RULE.

Divide the dividend by one of the component parts, or factors, and the quotient thence arising by the other, the last quotient will be the answer.

Note. The true remainder is found by multiplying the last remainder by the first divisor, and adding in the first remainder.

EXAMPLES.

1. Divide 138648 by 35.

7)138648	last rem. 1		5)138648
	×7	or	
5)19806—6	7		7)27729—3
	first rem.+6		
3961—1	true rem. 13		3961—2

True Quotient $3961\frac{13}{35}$.

2. Divide 148786 by 16. *Ans.* $9299\frac{2}{16}$.
3. Divide 487412 by 24 *Ans.* $20308\frac{20}{24}$.
4. Divide 964321 by 32. *Ans.* $30135\frac{1}{32}$.
5. Divide 87694 by 36. *Ans.* $2435\frac{34}{36}$.
6. Divide 144856 by 48. *Ans.* $3017\frac{40}{48}$.
7. Divide 846774 by 54. *Ans.* 15681.
8. Divide 80508 by 72. *Ans.* $1118\frac{12}{72}$.
9. Divide 1333260 by 108. *Ans.* 12345.
10. Divide 2642544 by 144. *Ans.* 18351.

To divide by 10, 100, 1000, *&c.*

RULE.

Cut off as many figures from the right hand of the dividend as there are ciphers in the divisor, and the figures cut off are the remainder, and the other figures of the dividend are the quotient.

EXAMPLES.

1. Divide 356 by 10. *Ans.* 35. rem. 6
2. Divide 6572 by 100. *Ans.* 65. rem. 72.
3. Divide 763735 by 1000. *Ans.* 763. rem. 735.

EXERCISES IN DIVISION.

1. If a man divide 4280 dollars equally among his five children; how many dollars will each receive? *Ans.* 856.

2. Suppose 125 acres of land cost 4125 dollars, how many dollars is that an acre? *Ans.* $33.

3. What number multiplied by 125, will make 17125? *Ans.* 137

4. If you were to divide 1875 dollars equally among 15 men, how many dols. would you give each one? *Ans.* $125.

5. If 24 hogsheads of molasses cost 456 dollars, what is that a hogshead? *Ans.* $19.

6. What number multiplied by 7, will make 2436? *Ans.* 348.

7. A drover purchased oxen, at $28 a head, to the amount of 6972 dollars; how many did he purchase? *Ans.* 249.

8. A prize of 294630 dollars is to be divided equally among 138 soldiers; how many dollars will each one have? *Ans.* 2135.

Questions.

1. What does Simple Division teach?
2. How many principal parts has division?
3. What is meant by dividend?
4. What is meant by divisor?
5. What is the quotient? Is the remainder greater, or less than the divisor?
6. What is Short Division? (Repeat the Rule.)
7. How do you prove division?
8. How may the remainder, after division, be expressed?
9. What is the Rule for Long Division?
10. When there are ciphers at the right hand of the diviser, how may we proceed?
11. When the divisor is a composite number, how may we proceed?
12. How is the true remainder found?
13. When the divisor is 10, 100, or 1000, &c., how would you proceed?
14. Write the sign of division on your slate.

REDUCTION OF FEDERAL MONEY.

The changing of numbers from one name to another without altering their value, is called Reduction.

EXAMPLES.

1. Reduce 148 dollars to cents and mills.

Thus, 14800 cents.
148000 mills.

To reduce dollars to cents, multiply by 100, because 1 dollar = 100 cents. That is, we annex 2 ciphers, and the whole will be cents. (*See Rule for multiplying* by 10, 100, *&c. page* 37.) And to reduce cents to mills, we annex 1 cipher, and the whole will be mills: therefore to change dollars to mills, we annex 3 ciphers.

2. In 1386 dollars 8 cents, how many cents?

Thus, 138608 cents. *Ans.*

Therefore, if the sum consists of dollars and cents, join them together as a whole number, and the whole will be cents. (In this example, the cents being less than 10, we write a cipher before them, or in the tens' place, which must always be done in like cases.) And, if the sum consists of dollars, cents and mills, join them together in like manner, and they will express so many mills.

Thus, 25 dollars, 41 cents, 5 mills = 25415 mills.

3. In 138608 cents, how many dollars and cents?

Thus, \$1386, 08 = 1386 dols. 08 cents. *Ans.*

To reduce cents to dollars, we divide by 100, because 100 cents = 1 dollar. (*See Rule for dividing by* 10, 100, *&c. page* 48.) That is, we cut off 2 figures to the right hand, and those on the left hand will be dollars; and to reduce mills to dollars, we point off three figures to the right; and those on the left will be dollars, and those on the right will be cents and mills.

Thus, 25415 mills = \$25,41,5, or 25 dols. 41 cts. 5 m.

4. Reduce 125 dollars to cents. *Ans.* 12500 cents.

5. Reduce 12550 cents to dollars. *Ans.*

6. Reduce 568 dollars, 9 cents, to cents.

Ans. 56809 cents.

7. Reduce 56809 cents to dollars. *Ans.*

8. Reduce 25 dollars, 58 cents, 8 mills to mills.
Ans. 25588 mills.

9. Reduce 25588 mills to dollars. *Ans.*

10. Reduce 450 dollars to mills. *Ans.* 450000 mills.

11. Reduce 3598 mills to dollars. *Ans.* $3,598.

12. Reduce 95410 cents to dollars. *Ans.* $954,10.

13. Reduce $96 12½ cents to mills. *Ans.* 96125 mills.

14. In 96125 mills, how many dollars and cents?
Ans. 96 dols. 12½ cts.

15. Write down 125 dols. and 9 mills as a whole number.
Ans. 125009 mills.

MULTIPLICATION OF FEDERAL MONEY.

RULE.

Multiply the given sum as in whole numbers, and place the separating point as many figures from the right hand in the product, as it is in the given sum, or multiplicand.

EXAMPLES.

1. Multiply 31 dollars 18 cents, by 25.

```
  $31,18
      25
  ------
   15590
   6236
  ------
```

Ans. 779,50 = $779 50 cts.

2. Multiply 21 dollars 8 mills, by 87.

```
  $21,008
       87
  -------
   147056
  168064
  -------
  1827,696
```

Ans. $1827, 69 cts. 6 mills.

			$	cts.	mls.
3.	Multiply $1 46 cts. 5 m. by 35.	*Ans.*	51	27	5
4.	Multiply 3 dols. 75 cts. by 98.	*Ans.*	367	50	
5.	Multiply $31, 98 c. 1 m. by 156.	*Ans.*	4989	03	6
6.	Multiply 81 dols. 5 cts. by 195.	*Ans.*	15804	75	
7.	Multiply $156, 28 c. 3 m. by 75.	*Ans.*	11721	22	5
8.	Multiply 13 dols. 5 m. by 29.	*Ans.*	377	14	5

9. Multiply 25 dols. 31¼ cts. or $253,15, by 365.
Ans. $9239,975.

To find the value of Goods, when the price of one yard, pound, &c. is given in Federal Money.

RULE.

Multiply the given price and quantity together, as whole numbers; then place the separatrix as many figures from the right hand in the product, as it is in the given price.

EXAMPLES.

1. What is the cost of 25 yards of broadcloth, at $4 58 cts. per yard?

```
$4,58 price of 1 yard.
   25 number of yards.
 ----
 2290
 916
 ------
 114,50=$114 50 cts. Ans.
```

2. What will 156 lbs. of coffee come to, at ,15 cts. per pound?

```
  156 number of lbs.
  ,15 price of 1 lb.
  ---
  780
 156
 ------
 $23,40=$23 40 cts. Ans.
```

3. What is the value of 18 barrels of flour, at $6,50 a barrel? *Ans.* $117.

4. What is the cost of 128 bushels of corn, at ,72 cts. a bushel? *Ans.* 92 dols. 16 cts.

5. What is the value of 25 lbs. of hyson tea, at $1,23 a pound? *Ans.* $30,75.

6. What cost 18 cwt. of rice, at 4 dols. 75 cts. per cwt.? *Ans.* 85½ dols.

7. What is the value of 25 pairs of shoes, at $1,75 a pair? *Ans.* $43,75.

8. Bought 65 gallons of molasses, at 27½ cents a gallon; what did the whole come to? *Ans.* $178,75.

9. What will 156 yards of Irish linen come to, at ,78 cents a yard? *Ans.* $121,68.

10. What will 18 days' labour come to, at ,96 cents a day? *Ans.* $17,28.

11. Bought 125 reams of paper, at $3,40 per ream; what did the whole come to? *Ans.* $425.

12. What will 125 bushels of oats come to, at ,34 cents a bushel? *Ans.* $42,50.

13. What cost 1258lbs. of cheese, at ,06 cents per pound? *Ans.* $75,48.

14. Find the value of 5 chests of tea, each weighing 64 pounds, at ,75 cents a pound. *Ans.* 240 dols.

15. A merchant bought 18 bales of cloth, each bale containing 25 pieces, and each piece 31 yards, at ,45 cts. a yard; what did the whole come to? *Ans.* $6277,50.

DIVISION OF FEDERAL MONEY.

RULE.

Write down the given sum, and divide as in whole numbers; the quotient will be the answer in the lowest name in the given dividend.

EXAMPLES.

1. If 3 yards of cloth cost 12 dols. 75 cts., what is that a yard?

Operation.

3)1275 cts.

Ans. 425=4 dols. 25 cts.

$12,75 is 1275 cts., which we divide by 3, as a whole number, and the quotient is 425 cents, which reduced to dollars, is 4 dollars 25 cents, the Answer.

2. Divide 21 dollars 6 cents equally among 12 men.

Operation.

12)2106,0

$ cts. mills

Ans. 1755 mills.=1,75,5

21 dols. 6 cts. is 2106 cents, which we divide by 12, and there is a remainder, which, by annexing a cipher, makes 60 mills, in which 12 is contained 5 times, and the quotient is 1755 mills; which being reduced to dollars, is 1 dollar 75 cents 5 mills.

3. Divide 230 dollars equally among 16 men.

```
   $
16)230(14,375
   16
   --
    70
    64
    --
    600
    48
    ---
    120
    112
    ---
      80
      80
      --
       0
```

We divide $230 by 16, and the quotient is $14, and a remainder of $6, which we reduce to cents by annexing 2 ciphers. We divide again, and the quotient is 37 cts. and the remainder 8 cents. We then reduce 8 cents to mills by annexing 1 cipher, and dividing again gives 5 mills. Thus the answer is 14 dollars 37 cents 5 mills.

But we may if we please, reduce $230 to mills = 230000 mills; then divide by 16, and the quotient will be 14375 mills; which reduced to dollars is = 14 dollars 37 cents 5 mills; the Answer.

4. Divide 5 dols. 9 cts, by 7.	*Ans.*	72 c. 7 m.+
5. Divide 6 dols. 93 cts. by 9.	*Ans.*	77 cents.
6. Divide 168 dols. 30 cts. by 85.	*Ans.*	$1,98 cts.
7. Divide 37 dols. 25 cts. by 298.	*Ans.*	$,125 m.
8. Divide $268 38 cts. by 135.	*Ans.*	$1,98,8 m.
9. Divide 225 dols. by 900.	*Ans.*	$,25 cts
10. Divide 155 dols. by 301.	*Ans.*	51 cts. 4 m.+

PRACTICAL QUESTIONS.

1. Bought 12 barrels of flour for 78 dollars; what was that a barrel? *Ans.* $6, 50.

2. If 26 gallons of molasses cost $11,44, what is that a gallon? *Ans.* ,44 cents.

3. Bought 25 lbs. of coffee for 4 dollars; how much is that a pound? *Ans.* ,16 cents.

4. If 112 lbs. of sugar cost $10,64, how much will 1 pound cost at that rate? *Ans.* $9\frac{1}{2}$ cents.

5. If 18 gallons of brandy cost $42,75, what is that a gallon? *Ans.* $2,375, $2,37$\frac{1}{2}$.

6. If a man's wages be $254,04 a year, what is that a calendar month? *Ans.* $21,17.

7. A prize of 1562 dollars 55 cents is to be divided equally among 125 sailors; what will each one receive? *Ans.* $12,50+.

8. If a man's salary be 3650 dollars in a year, or 52 weeks, what is that a week? *Ans.* $70 19 cts. 2 m.

9. Bought 125 yards of cloth for $181,25; what did it cost a yard? *Ans.* $1,45.

SUPPLEMENT TO MULTIPLICATION.

To multiply by a mixed number; that is, a whole number joined with a fraction, as $5\frac{1}{2}$, $6\frac{1}{4}$, $8\frac{3}{4}$, $7\frac{1}{3}$, &c.

RULE.

Multiply by the whole number, and take $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{1}{3}$, &c., of the multiplicand, and add it to the product.

Note. To get $\frac{1}{4}$ of a number, divide by 4; $\frac{1}{2}$, divide by 2; $\frac{1}{3}$, divide by 3; $\frac{1}{5}$, divide by 5; $\frac{1}{6}$, divide by 6, &c.

1. Multiply 47 by $18\frac{1}{2}$.

Operation.

$$
\begin{array}{rr}
 & 2)47 \\
 & 18\frac{1}{2} \\
\hline
\frac{1}{2}\text{ Multiplicand} = & 23\frac{1}{2} \\
 & 376 \\
 & 47 \\
\hline
\text{Product} & 869\frac{1}{2}
\end{array}
$$

To multiply by $\frac{1}{2}$, we take $\frac{1}{2}$ of the multiplicand; that is, divide by 2. Thus, $\frac{1}{2}$ of 47 is $23\frac{1}{2}$.

We then multiply by 18, and add the several products together, and the answer is $869\frac{1}{2}$.

2. Multiply 56 by $9\frac{3}{4}$.

$$
\begin{array}{rr}
 & \frac{2}{4})56 \\
 & 9\frac{3}{4} \\
\hline
\frac{1}{2}\text{ of Multiplic.} & =28 \\
\frac{1}{4}\text{ of Multiplic.} & =14 \\
 & 504 \\
\hline
\text{Answer} & 546
\end{array}
$$

Obs. $\frac{3}{4}=\frac{1}{2}$ and $\frac{1}{4}$; therefore we take the multiplicand $\frac{1}{2}$ a time and $\frac{1}{4}$ of a time, which are equal to $\frac{3}{4}$ of the multiplicand.

We then multiply by 9, and add the products together, which gives the answer.

3. Multiply 46 by $12\frac{1}{3}$.

$$
\begin{array}{rr}
 & 3)46 \\
 & 12\frac{1}{3} \\
\hline
\frac{1}{3}\text{ of Multiplic.} & =15\frac{1}{3} \\
 & 552 \\
\hline
\text{Ans.} & 567\frac{1}{3}
\end{array}
$$

4. How much is $\frac{5}{8}$ of 96?

$$
\begin{array}{rcr}
\text{Thus } 8)96 & \quad & \text{or thus } 96 \\
\frac{1}{8}=12 & & 5 \\
\times 5 & & \overline{8)480}= \\
\hline
\textit{Ans. } 60 & & 60
\end{array}
$$

In the first, we take 5 times $\frac{1}{8}$ of 96. In the second place, we take $\frac{1}{8}$ of 5 times 96: either of which methods may be practised.

5. Multiply 312 by $30\frac{1}{2}$. *Ans.* 9516.
6. Multiply 334 by $9\frac{1}{3}$. *Ans.* $3117\frac{1}{3}$.
7. Multiply 511 by $19\frac{1}{5}$. *Ans.* $9811\frac{1}{5}$.
8. Multiply 6597 by $5\frac{1}{7}$. *Ans.* $33927\frac{3}{7}$.
9. Multiply 1825 by $7\frac{1}{8}$. *Ans.* $13003\frac{1}{8}$.
10. Multiply 3462 by $4\frac{1}{6}$. *Ans.* 14425.

EXAMPLES FOR PRACTICE.

1. What will $15\frac{1}{2}$ acres of land come to, at \$37,50 per acre? *Ans.* \$581,25.

2. In 36 barrels of beans, each barrel containing $3\frac{1}{2}$ bushels, how many bushels? *Ans.* 126.

3. It takes 320 rods to make a mile, and $5\frac{1}{2}$ yards to make a rod; how many yards in a mile? *Ans.* 1760.

4. Sold a ship for \$11341,52; I owned $\frac{3}{8}$ of her; what was my part of the money? *Ans.* \$4253,07 cts.

5. What will 235$\frac{1}{2}$ acres of land cost, at \$48,50 per acre? *Ans.* \$11421,75

6. What will 156 lbs. of Indigo come to, at 2$\frac{1}{2}$ dollars per pound? \$*Ans.* 390.

7. What will 12$\frac{5}{8}$ yards of broadcloth cost, at \$4,32 per yard? *Ans.* \$54,54.

8. What cost 5$\frac{1}{3}$ yards of satin, at 2 dollars 35 cts. per yard? *Ans.* \$12,53$\frac{1}{3}$.

9. Bought 48 pieces of cloth, each piece containing 31 yards, at 37$\frac{1}{2}$ cents per yard; what did the whole come to? *Ans.* \$558.

A CONCISE METHOD

To find the cost of articles, when the price is an aliquot, or even part of a dollar, (usually called Practice.)

The aliquot part of any number is such a part of it, as being taken a certain number of times, exactly makes that number.—Thus, 25 cts. is an aliquot, or even part of a dollar, because 4 times 25 cents is just equal to 1 dollar.

A TABLE OF ALIQUOT, OR EVEN PARTS.

50 cents $=\frac{1}{2}$ of a dollar.	12$\frac{1}{2}$ cents $=\frac{1}{8}$ of a dollar.
33$\frac{1}{3}$ cents $=\frac{1}{3}$ of a dollar.	10 cents $=\frac{1}{10}$ of a dollar.
25 cents $=\frac{1}{4}$ of a dollar.	8$\frac{1}{3}$ cents $=\frac{1}{12}$ of a dollar.
20 cents $=\frac{1}{5}$ of a dollar.	6$\frac{1}{4}$ cents $=\frac{1}{16}$ of a dollar.
16$\frac{2}{3}$ cents $=\frac{1}{6}$ of a dollar.	5 cents $=\frac{1}{20}$ of a dollar.

RULE.

Divide the given quantity by the number of even parts in the price, which it takes to make 1 dollar. Thus, if the price be 50 cents, divide bv 2; if 25 cents, or $\frac{1}{4}$ of a dollar, divide by 4, &c.

EXAMPLES.

1. What is the value of 14765 yards of cloth at 25 cents or $\frac{1}{4}$ of a dollar per yard?

$$\begin{array}{r} 4)14765,00 \\ \hline 3691,25 \end{array}$$

25 cts.$=\frac{1}{4}$ of a dollar, therefore we divide the number of yards by 4; and to the remainder we annex two ciphers and divide again, which gives cents. *Ans.* \$3691,25

2. What will 4764 lbs. of butter come to, at 12½ cents per pound? *Ans.* $595,50.

3. What cost 4680 bushels potatoes, at 25 cents per bushel? *Ans.* 1170.

4. What cost 2460 bushels of rye at 50 cents a bushel? *Ans.* $1230.

5. What cost 9366 yards of cloth at 33⅓ cents, or ⅓ of a dollar per yard? *Ans.* $3122.

6. At 16⅔ cents a yard, what will 9762 yards of ribbon come to? *Ans.* $1627

7. What cost 1768 yards, at $1,12½cts. a yard?

Thus, 8)1768=cost at 1 dollar a yard.

221=cost at 12½cts. or ⅛ of a dollar a yard.

Ans. $1989=cost at $1,12½ cents a yard.

8. What is the value of 1245 bushels of wheat at $1,50 per bushel? *Ans.* $1867,50.

9. What will 127lbs. of indigo come to, at $4,33⅓ cents per pound? *Ans.* $550,33,3+

Questions.

1. How do you change dollars to cents?
2. How do you change cents to dollars?
3. How are dollars reduced to mills? mills to dollars? cents to mills? mills to cents?
4. How do you multiply Federal Money?
5. How do you find the value of goods when the price of 1 yard, &c. is given in Federal Money?
7. How do you divide the denominations of Federal Money
7. How do you multiply by a mixed number?
8. By what number would you divide to get one half of any sum? to get one third? one fourth? one fifth? &c.
9. How do you find the cost of articles when the price is an aliquot or even part of a dollar?
10. What part of a dollar is 25 cents? is 50 cents? is 33 1-3 cents? is 16 2-3 cents? is 12 1-2 cents? is 6 1-4 cents? is 10 cents? is 20 cents? is 5 cents? is 8 1-3 cents? is 4 cents?

EXERCISES IN FEDERAL MONEY.

1. Bought a hogshead of molasses containing 65 gallons, at 27 cents per gallon; and 15 barrels of flour, at $6,50 per barrel: what did the whole amount to? *Ans.* $115,05.

2. Bought 9 yards of broadcloth, at $5,43 per yard; 12 yards of linen, at 58 cents per yard; 23 yards cotton cloth, at 12½ cents per yard; and a book for 68 cents; what did the whole amount to? *Ans.* $59,38¼

3. Bought 3 hogsheads of sugar, each weighing 650lbs. at 9½ cents per pound; what did the whole cost?

Ans. $185,25.

4. A farmer sold 36 bushels of corn, at 75 cents a bushel; 25 bushels of rye, at 82 cents a bushel; and 145 bushels of potatoes, at 25 cents a bushel; and received $51,75 in part payment: how much remains due? *Ans.* $32.

5. How many yards of cloth at $3 per yard, may be had for 136 bushels of wheat at $2 per bushel?

Ans. 90⅔ yards.

Find the amount of the following.

BILLS.

1. Merchant's Bill.

New London, June 20th, 1836.

Mr. Lewis Simmons,

Bought of William Merchant.

8 yds. black broadcloth,	at	$4,50 per yard,
25 yds. cotton cloth,	at	,12½ "
27 lbs. hyson tea,	at	$1,25 per lb.
56 lbs. coffee,	at	,20 "
25 pair men's shoes,	at	1,75 per pair,
28 yds. of calico,	at	,33⅓ per yard,
1 hhd. sugar, 8½ cwt.	at	9,50 per cwt.
36 bushels of oats,	at	,33⅓ per bushel.
Received Payment,		$229,90,8+

William Merchant.

2. Farmer's Bill.

Mr. James Paywell,

To Andrew Silliman, *Dr.*

1836					
Nov. 10,	To 156	bushels corn,	at $	,75	per bushel,
"	" 37	" wheat,	at	1,50	"
12,	" 97	" oats,	at	,33⅓	"
25,	" 347	" potatoes,	at	,25	"
"	" 95	" beans,	at	1,12½	"
Dec. 6,	" 19¾	tons hay,	at	14,75	per ton.
Dec. 25.	*Received Payment,*				$689,77,0+

Andrew Silliman.

COMPOUND NUMBERS.

Any number expressing things in the same name or kind, is called a simple number; as 60 men, 45 years, &c.

But when a number expresses things of different names or kinds, it is called a Compound number; as 56 pounds 4 shillings and 6 pence; 3 years 4 months 25 days; 5 miles 3 furlongs, &c.

Different names or kinds are called different denominations.

COMPOUND ADDITION.

Is the adding of several numbers together, having different denominations.

RULE.

1. Place the numbers so that those of the same name or denomination may stand directly under each other.

2. Add the first column, or lowest denomination, as in whole numbers; then divide the sum by as many of the same denomination as make one of the next greater, setting down the remainder under the column added, and carry the quotient to the next superior denomination; continuing the same to the last, which add as in simple addition.

I. *Sterling Money.*

In England, Money is reckoned in pounds, shillings, pence, and farthings, which is called Sterling Money.

TABLE.

4 farthings (qrs.)	make 1 penny, marked d.
12 pence	make 1 shilling, " s.
20 shillings	make 1 pound, " £

Note.—Farthings are often written as the fraction of a penny; thus, 1 farthing is written $\frac{1}{4}$d., 2 farthings, $\frac{1}{2}$d., 3 farthings, $\frac{3}{4}$d.

EXAMPLES.

1. Add together £74 13s. 6$\frac{1}{4}$d.; £18 5s. 7d.; £24 10s. 9$\frac{3}{4}$d.; and £13 8s. 4$\frac{1}{2}$d.

Operation.			
	20	12	4
£	s.	d.	qr.
74	13	6	2
18	5	7	0
24	10	9	3
13	8	4	2
130	18	3	3

Illustration.—We find by adding up the column of qrs. the amount is 7, which we divide by 4, because 4 qrs.=1 penny, and the quotient is 1 penny, and the remainder is 3qrs.; we set down the 3qrs. under the column of qrs. and carry the quotient (1d.) to the column of pence, and the amount is 27d. which we divide by 12, the number of pence in a shilling, and the quotient is 2s. and the remainder 3d.; we place the remainder 3d. under the column of pence, and carry the quotient to the shillings, and the amount is 38s. which divided by 20, gives the quotient £1, and the remainder 18s. which we place under the shillings, and carry the 1 to the pounds, and the amount is £130.

£	s.	d.
125	13	$4\frac{1}{4}$
94	6	$10\frac{1}{4}$
85	19	6
64	17	$9\frac{3}{4}$
370	17	$6\frac{1}{2}$

£	s.	d.	qrs.
72	9	6	2
118	10	11	3
95	17	9	2
25	14	4	2

£	s.	d.	qrs.
12	14	9	3
17	13	8	1
8	9	0	0
	7	3	3

4. Add £95 13s. 4d., £23 16s. 9d., £27 0s. 10d., £13 18s. and £15 1s. 6d. together. *Ans.* £175 10s. 5d.

6. Find the whole amount of £10 10s. 10d. 3qrs., 9s. 6d. 2qrs., 7s. 3d. 1qr. and $11\frac{3}{4}$d. *Ans.* £11 8s. $8\frac{1}{4}$d.

II. *Troy Weight.*

lb.	oz.	pwt.	gr.
14	11	13	23
8	4	16	19
7	8	13	16
6	10	14	9
3	8	10	7
	7	11	12
42	4	0	14

lb.	oz.	pwt.	gr.
1	8	3	11
2	10	13	10
7	6	5	13
3	11	10	9
1	9	11	18
5	10	8	16

lb.	oz.	pwt.	gr.
1	11	10	11
	10	13	14
2	6	9	9
3	5	18	16
1	8	12	19
8	11	10	13

4. Add together the following quantities of silver, viz.: 3lb. 9oz. 16pwt. 3gr., 10oz. 1pwt. 22gr., 3lb. 4oz. 0pwt, 6gr., and 5lb. 3oz. 13pwt. 3gr.

Ans. 13lb. 3oz. 11pwt. 10gr.

5. A goldsmith bought 4 ingots of silver, the first weighed 7 lb 2oz. 12pwt., the second 6lb. 4oz. 5pwt. 6gr.,

he third 8lb. 3pwt. 1gr, the fourth 6lb. 11oz. 5pwt 14gr. What quantity did he buy in all?

Ans. 28lb. 6oz. 5pwt. 21gr.

III. *Avoirdupoise Weight.*

cwt.	qr.	lb.
21	3	25
19	2	14
12	1	10
9	0	27
63	0	20

cwt.	qr.	lb.	oz.
1	3	16	14
3	0	12	10
5	1	9	13
7	2	27	15

T.	cwt.	qr.	lb.	oz.	dr.
14	11	1	15	6	10
24	0	3	11	14	15
7	19	0	25	9	8
3	11	2	14	6	13

4. A grocer sold 4hhds. of sugar. The first weighed 7cwt. 1qr. 16lb., the second 6cwt. 2qr. 10lb., the third 9cwt. 2qr. 14lb., the fourth 7cwt 0qr. 10lb.; what was the weight of the whole? *Ans.* 30cwt. 2qr. 22lb.

5. A farmer sold 3 loads of hay, weighing as follows viz.: the first 19cwt. 3qr. 21lb., the second 21cwt. 1qr. 14lb., the third 18cwt. 2qr.; what was the weight of the whole?

Ans. 59cwt. 3qr. 7lb.

6. Add together 2T. 1cwt. 3qr., 19cwt. 1qr. 12lb., 14cwt. 3qr. 18lb., 2cwt. 3qr., 15cwt. 1qr. 19lb., 5T. 6cwt., 18cwt. 0qr. 13lb. 15oz., and 27lb. 15oz.

Ans. 10T. 18cwt. 2qr. 6lb. 14oz.

IV. *Apothecaries Weight.*

ʒ	℈	gr.
9	1	17
3	2	19
6	1	17
5	2	12
8	1	10
34	1	15

℥	ʒ	℈	gr.
10	3	2	19
6	3	0	12
7	6	1	17
9	5	2	12
6	1	0	16

℔	℥	ʒ	℈	gr
5	9	3	2	13
4	8	6	0	19
9	10	5	2	12
6	5	6	1	17
8	9	4	0	9

V. *Cloth Measure.*

yds.	qr.	na.
36	2	1
43	1	3
28	3	2
15	1	1
65	0	0
189	0	3

EE.	qr.	na.
91	3	2
49	4	3
6	2	1
61	1	0
15	0	1

E.Fl.	qr.	na.
41	2	1
39	1	3
18	0	2
27	2	1
9	1	0

E.Fr.	qr.	na.
75	5	2
13	1	0
47	3	3
27	4	1
115	1	2

5. There are 5 pieces of cloth which measure as follows, viz.: 35yds. 2qr. 1na., 28yds. 3qr., 37yds. 3qr. 3na., 33yds. 3qr. 3na., 41yds. and 3na.; how many yards in the whole?
Ans. 177yds. 1qr. 2na.

6. Add together 151yds. 2qr., 75yds. 0qr. 1na., 45yds., 56yds. 1qr., 13yds. 3qr. 1na., 29yds. 2qr. and 95yds. 1qr, 3na. Ans. 466yds. 2qr. 1na.

7. A merchant bought 4 pieces of broadcloth; the first contained 19yds. 3qr., the second 15yds. 2qr. 1na., the third 25yds., the fourth 15yds. 1qr. 3na.; how many yards did he buy? Ans. $75\frac{3}{4}$yds.

8. Find how much cloth is contained in the 3 following pieces; the first $47\frac{3}{4}$yds., the second 43yds. 3na., and the third 26yds. 1qr. 3na.? Ans. 117yds. 1qr. 2na.

VI. *Dry Measure.*

bu.	pk.	qt.
35	3	2
19	1	7
15	0	2
7	2	5
5	1	3
83	1	3

bu.	pk.	qt.	pt.
115	3	1	1
143	0	7	0
74	2	6	1
16	1	3	1
27	3	0	0

bu.	pk.	qt.	pt.
157	3	6	1
176	1	7	0
113	3	4	0
114	2	3	1
106	0	6	1

4. Add together 3bu. 3pk. 1qt., 1bu. 7qts., 16bu. 1pk., 21bu. 3pk. 4qts., 13bu. 1pk. 2qts, and 18bu. 3pks.
Ans. 75bu. 0pk. 6qts.

5. Bought 6bu. 3pks, of wheat at one time, 3bu. 3pks. at another time, 18bu. 1pk. at another, and 19bu. 2pks. 1qt, at another time; what quantity did I buy in all?
Ans. 48bu. 1pk. 1qt.

VII. *Wine Measure.*

gal.	qt.	pt.
36	3	1
17	1	1
24	3	1
15	2	1
47	1	1

hhd.	gal.	qt.	pt.
42	34	3	1
27	36	2	1
24	24	1	0
16	6	3	0
5	0	1	1

T.	P.	hhd.	gal.	qt.	pt.	gi.
24	1	1	34	2	1	3
17	0	0	15	3	0	2
10	1	1	16	2	1	0
6	1	0	62	1	1	1
7	0	1	53	3	1	3

4. Add together 28gals. 2qts. 1pt. 2gi., 16gal. 1qt. 2gi., 25gal. 1qt. 1pt. 2gi., 16gal. 1qt., 12gal. 3qts., and 35gal. 2qts. 1pt. *Ans.* 135gal. 2gi.

5. A merchant bought 3 casks of brandy containing as follows, viz.: 71 gallons 3 quarts, 65 gallons 3 quarts, 67 gallons 1 quart; how many gallons did he buy?

Ans. 204 gallons 3 quarts.

6. Bought 7 casks of molasses, containing as follows; 76 gallons 3 quarts 1 pint, 65¾ gallons, 46 gallons 1 pint, 57 gallons 1 quart, 63 gallons, 65 gallons 2 quarts 2 pints, and 64 gallons 1 pint; how much in the whole?

Ans. 438 gallons 3 quarts 1 pint

VIII. *Long Measure.*

yds.	feet.	in.	b.c.
4	2	11	2
3	1	8	1
1	0	6	1
5	2	9	0
2	1	10	2
18	0	10	0

m.	fur.	rd.
45	4	26
36	5	23
24	7	34
11	3	7
16	1	33

le.	m.	fur.	rd.
76	2	6	32
54	1	5	26
64	1	7	24
46	2	4	23
23	2	3	26

4. Add together 27 mils 3 furlongs, 35 miles 1 furlong 26 poles, 30 miles 4 furlongs 15 poles, 45 miles, and 84 miles 7 furlongs. *Ans.* 223m. 1rd.

5. If a man travel 6 days at the rate of 29 miles 2 furlongs 10 poles each day, how far will he travel in all?

Ans. 175m. 5fur. 20rd.

IX. *Land or Square Measure.*

A.	roods.	rd.
467	3	31
623	2	17
31	1	27
64	3	33

A.	roods.	rd.
756	3	36
374	1	14
60	2	6
7	0	37

sq.ft.	sq. in.
245	128
93	138
128	125
13	56

4. A man has 3 fields which measure as follows, viz.: 17 acres 2 roods 18 poles, 39 acres 1 rood 16 poles, 12 acres 0 roods 36 poles; how much land in the 3 fields?

Ans. 69 acres 0qr. 30rd.

5. Add together 16 acres 1 rood 3 rods, 14 acres 3 roods 39 rods, 48 acres 1 rood 17 rods, 65 acres 3 roods 28 rods, and 19 acres 1qr. 25 rods.

Ans. 164 acres 3 roods 32 rods.

6. Find the number of acres in the following tracts of land, viz.: the first contains 489 acres 3 roods 28 rods, the second 276 acres 1 rood 30 rods, the third 596 acres 2 roods 18 rods, the fourth 309 acres 3 roods 34 rods.

Ans. 1672 acres 3 roods 30 rods.

X. *Solid or Cubic Measure.*

Tons.	feet.
41	38
19	25
7	11
9	16
78	10

cords.	feet.
6	127
9	118
5	10
7	89

cords.	feet.	inches.
14	111	1446
9	84	1726
5	127	96
10	76	1236

4. A man bought 3 parcels of wood; the first contained 2 cords 84 feet 864 inches; the second, 5 cords 36 feet 1456 inches; and the third, 96 cords 84 feet. How much did he buy in all? *Ans.* 104 cords 77 feet 592 inches.

5. Add together 25 cords 18 feet, 128 inches; 19 cords 32 feet; 14 cords 13 feet 1600 inches; and 125 cords 64 feet. *Ans.* 184 cords.

XI. *Time.*

da.	h.	m.	sec.
57	23	55	40
34	8	40	17
19	6	10	11
8	17	34	48

yr.	mo.	w.	da.	h.	m.	sec
8	6	3	6	14	56	30
1	3	0	5	23	59	59
5	1	2	1	18	15	40
7	0	1	4	6	0	45

3. Add together 16 yrs. 154 d. 13 h.; 24 yrs. 94 d. 18 h.; 94 yrs. 184 d. 23 h.; 56 yrs. 364 d. 5 h.; and 25 yrs. 14 d. 1 h. *Ans.* 217 yrs. 82 d. 12 h.

4. Add together 5 yrs. 125 d. 18 h.; 5 yrs. 19 d. 6 h.; 17 yrs. 156 d. 5 h.; 356 d.; 18 h.; 87 yrs.; and 23 yrs. 134 d. 6 h. *Ans.* 139 yrs. 62 d. 5 h.

XII. *Circular Motion.*

S.	°	′	″		S.	°	′	″
6	29	40	20		11	13	45	59
3	5	59	48		9	29	18	00
1	18	27	30		5	18	27	1
2	11	45	20		3	5	18	27

3. Add together 345 deg. 25 m. 30 sec.; 145° 18′ 30″; 85° 59′ 48″; 45° 38′ 44″; and 30° 15′ 19″.

Ans. 652° 37′ 51″.

COMPOUND SUBTRACTION.

Teaches to find the difference between any two numbers of diverse denominations.

RULE.

I. Place those numbers under each other which are of the same denomination, the less being below the greater.

II. Begin with the lowest denomination, and if it exceed the number above it, borrow as many units as make one of the next higher denomination; subtract it therefrom, and to the difference add the upper number, remembering always to add one to the next superior denomination for that which you borrowed.

Proof.—The method of Proof is the same as Simple Subtraction.

EXAMPLES.

Sterling Money.

1. Borrowed £30 4s. 9d., and paid £5 14s. 6d.; how much remains unpaid?

Operation.

	£	20 s.	12 d.
Minuend	30	4	9
Subtrahend	5	14	6
Remains unpaid	24	10	3

Illustration.—We write the numbers, the less below the greater, pence under pence, shillings under shillings, &c. We take 6d. from 9d., and the remainder is 3d. Proceeding to the shillings, we cannot take 14s. from 4s., but we borrow 1 from the pounds =20s., and 14s. subtracted from

20s. leave 6s., to which we add the upper number, 4s. making 10s., which we set down. Proceeding to the pounds, we carry 1 to £5 makes £6, which subtracted from £30 leave £24, and the work is done.

	£	s.	d.	£	s.	d.	£	s.	d.	qr.
From	18	16	4	125	19	3	165	14	6	1
Take	8	13	6	112	16	2	131	19	4	3
	10	2	10							

6. An English merchant sold goods to the amount of £145 13s. 6½d., and received in part payment £55 18s 4¾d.; how much remained due, or unpaid.

Ans. £89 15s. 1d. 3qrs

Troy Weight.

lb.	oz.	pwt.	grs.	lb.	oz.	pwt.	grs.
87	11	11	13	27	10	15	22
19	11	14	22	15	9	16	23
67	11	16	15				

3. From 2lb. 11oz. 13pwt. take 1lb. 0oz. 19pwt.

Ans. 1lb. 10oz. 14pwt.

4. A silversmith having 5lb. 3oz. 13pwt. of silver, worked up 11oz. 14pwt. 16grs. of it; how much had he left?

Ans. 4lb. 3oz. 18 pwt. 8grs.

Avoirdupoise Weight.

cwt.	qr.	lb.	T.	cwt.	qr.	lb.	oz.	dr.
59	1	11	100	10	1	11	13	14
19	3	27	15	13	1	18	12	15

3. From 8T. 13cwt. take 7cwt. 1qr. 14lb. 11oz.

Remain 8T. 5cwt. 2qr. 13lb. 5oz.

4. A merchant bought a hogshead of sugar, containing 9cwt. 2qrs., and sold out of it 5cwt. 3qrs. 16lb. 8oz.; how much remained unsold? *Ans.* 3cwt. 2qrs. 11lb. 8oz.

Apothecaries Weight.

lb.	oz.	dr.	ʒ	℈	gr.	lb.	℥	ʒ	℈	gr.
19	8	6	4	1	17	35	7	4	1	14
9	11	5	1	2	13	17	10	7	1	13

Cloth Measure.

yds.	qr.	na.		E. E.	qr.	na.		E. Fl.	qr.	na.
35	1	2		457	3	1		756	1	3
21	1	3		291	3	2		149	2	1

4. Bought a piece of broadcloth, containing 29 yards 3 quarters 1 nail, and sold 9 yards 1 quarter 3 nails off of it; how much remained unsold? *Ans.* 20yds. 1qr. 2na.

Dry Measure.

bu.	pk.	qt.		bu.	pk.	qt.		bu.	pk.	qt.	pt.
95	1	7		18	1	5		17	2	3	0
15	3	1		13	1	6		7	1	6	1

4. A farmer raised 96 bushels of wheat, and sold 36 bushels 3 pecks 4 quarts out of it; how much had he remaining? *Ans.* 59bu. 0pk. 4qts.

5. Subtract 15 bushels 3 pecks 2 quarts 1 pint from 95 bushels, and how much will remain?

Ans. 79bu. 5qts. 1pint.

Wine Measure.

gal.	qt.	pt.	gi.		hhd.	gal.	qt.	pt.		T.	hhd.	gal.	qt.	pt.
21	2	1	1		13	0	1	0		5	3	20	3	1
15	2	1	3		10	59	3	1		3	2	27	0	0

4. From 13hhds. 15 gallons, take 11hhds. 20 gallons 3 quarts. Remain 1hhd. 57gal. 1qt.

5. From a hogshead of molasses, containing 63 gallons, a grocer sold 18 gallons 3 quarts 1 pint; how much remained in the hogshead? *Ans.* 44gals. 1pt.

Long Measure.

yd.	ft.	in.	bc.		m.	fur.	po.		le.	m.	fur.	po.
5	2	11	0		45	6	22		96	2	6	33
3	2	11	1		11	6	23		24	1	7	32

4. From 36 miles 5fur. 26 rods, take 15 miles 1 furlong 36 poles. *Ans.* 21m. 3fur. 30 poles.

5. The distance from New London to Hartford is 42 miles. If a man start from New London and travel towards Hartford 18m. 5fur. 35 rods, how far will he then be from Hartford? *Ans.* 23m. 2fur. 5 rods.

Land or Square Measure.

A.	roods.	r.
39	2	10
24	1	26

A.	r.	po.
45	2	17
26	1	36

sq. ft.	sq. in.
339	141
29	142

4. From 17 acres 3qr. 35 rods, subtract 12 acres 2 roods 14 rods. Remain 5A. 1 rood 21rods.

5. If from an enclosure of 45 acres 3 roods and 26 po., there be taken 21 acres 2 roods 39 poles, how much will there be remaining? *Ans.* 24A. 0qr. 27rds.

Solid or Cubic Measure.

Tons.	feet.
114	26
109	39

cords.	feet.
82	113
31	120

Tons.	feet.	in.
48	16	140
15	14	145

4. From 2 tons 38 feet of hewn timber, take 1 ton 46 feet 1536 inches. Remain 41 feet 192in.

5. If from a parcel of wood containing 34 cords 96 feet, there be sold 28 cords 32 feet, how much will be left? *Ans.* $6\frac{1}{2}$ cords.

Time.

yr.	mo.	w.	da.
45	11	3	1
36	10	3	5

yr.	da.	h.	m.	sec.
34	359	19	41	20
18	355	18	49	19

3. From 94 years 134 days, subtract 25 years 156 days 5 hours 40 minutes. Remain 68yrs. 342da. 18h. 20m.

4. William's age is 16 years 3 months and 20 days, and

Henry's is 14 years 6 mo. 19 days; can you tell me how much older William is than Henry? *Ans.* 1yr. 9mo. 1da.

Circular Motion.

S.	°	′	″		S.	°	′	″
7	25	46	45		11	29	34	54
3	9	40	56		7	29	51	37

3. The whole circumference of the earth is 360 degrees. If a ship could circumnavigate the globe by a direct course, how much farther must she sail to complete her voyage, after sailing 248° 55′ 31″? *Ans.* 111° 04′ 29″.

EXERCISES IN COMPOUND ADDITION AND SUBTRACTION.

1. Borrowed £62 10s.; paid at one time £14 16s. $8\frac{1}{4}$d., and at another time £21 10s.; how much remains unpaid? *Ans.* £26 3s. $3\frac{3}{4}$d.

2. Bought a hogshead of sugar, weighing 8cwt. 3qrs. 14lb.; sold at one time 3cwt. 1qr. 14lb. 5oz., and at another time 3qrs. 16lb. 12oz. How much remained unsold? *Ans.* 4cwt. 2qr. 10lb. 15oz.

3. From a piece of broadcloth, containing 56 yards 2 nails, a tailor made 3 suits, each 6 yards 3 qrs.; how much remained in the piece? *Ans.* 35yds. 3qrs. 2na.

4. A merchant bought a hogshead of molasses, containing 65 gallons, and sold at one time 14 gallons 3 quarts, and at another time 9 gallons 2 quarts, 1 pint; how much remained in the hogshead? *Ans.* 40gal. 2qt. 1pt.

5. A man owning a farm of 321 acres 2 quarters, sold one field containing 14 acres 3 roods 19 poles, and another containing 12 acres 1 rood; how much land had he left? *Ans.* 294A. 1qr. 21 rods.

6. From a pile of wood, containing 25 cords, there was sold at one time 10 cords 84 feet, and at another time 6 cords 84 feet; how many cords were left? *Ans.* 7cds. 88ft.

7. The distance from Philadelphia to Washington is 138 miles; if a man travel 5 days from Philadelphia towards Washington, viz.: the first day 18 miles 3fur. 25 rods; the

second, 23m. 2fur.; the third 20m. 1fur. 6 rods; the fourth 25m., and the fifth day, 22m. 3fur. 18 rods; how far will he then be from Washington? *Ans.* 28m. 5fur. 31 poles.

To find the length of Time from one date to another.

RULE.

Subtract the first date from the last date, reckoning the months according to their order; and in computing interest on Notes, &c., each month is reckoned 30 days.

EXAMPLES.

1. What is the difference in time from February 19th, 1827, to June 19th, 1833?

Operation.

	yrs.	mo.	da.
Last date	1833	5	19
First date	1827	1	19
Answer	6	4	0

Illustration. In reckoning from January 1st, to June 19th, inclusive, we find it to be 5 mo. 19 days, and from January 1st to February 19th, is 1 month 19 days; and the first date subtracted from the last date, leaves 6 years 4 months 0 days, the answer.

2. What is the difference of time from July 12, 1829, to January 27, 1834? *Ans.* 4yrs. 6mo. 15da.

3. A Note, bearing date July 15, 1830, was paid November 20, 1833; how long was the note at interest?
Ans. 3 years 4 months 5 days.

4. A Note, bearing date December 10, 1828, was paid January 5, 1834; how long was it at interest?
Ans. 5 years 0 months 25 days.

5. The war between England and America commenced April 19, 1775, and peace took place January 20, 1783; how long did the war continue?
Ans. 7 years 9 months 1 day.

Questions.

What is a compound number?
What is Compound Addition?
How do you write numbers to be added?
How do you add the numbers?
What does Compound Subtraction teach?
How do you place the numbers?
How do you subtract? (Repeat the Rule.)
How do you find the length of time from one date to another?

COMPOUND MULTIPLICATION.

Compound Multiplication is when the multiplicand consists of diverse denominations.

I. *When the Multiplier does not exceed* 12.

RULE.

Write down the multiplicand and place the quantity under the least denomination for a multiplier. Multiply each denomination separately, and carry as in compound addition, setting down the whole product of the highest denomination.

EXAMPLES.

1. What will 6 yards of cloth cost at £1 5s. 7¾d. per yard?

Operation.

	£	s.	d.	qr.	
	1	5	7	3	=price of 1 yard.
				6	=number of yards.
Ans.	7	13	10	2	=cost of six yards.

Illustration. 6 times 3qrs. are 18qrs., which we divide by 4 (as in compound addition,) and the remainder is 2qrs. which we set under the qrs., and the quotient is 4d. which we carry, saying 6 times 7d. are 42d. and 4 are 46d. which divided by 12 (the number of pence in a shilling,) the remainder is 10d. and the quotient 3s., then 6 times 5s. are 30s. and 3s. to carry make 33s., which divided by 20 the remainder is 13s. and the quotient is £1, and 6 times £1 is £6, and £1 to carry makes £7, which set down.

2. Multiply 11s. 4d. by 3. 3. Multiply £1 11s. 9½d. by 5.

	£	s.	d.
	0	11	4
			3
Ans.	1	14	0

	£	s.	d.	qrs.
	1	11	9	2
				5
Product,	7	18	11	2

4. Multiply £539 14s. 10d. 3qrs. by 5.

Ans. £2698 14s. 5¾d

5. Multiply £50 18s. 10d. by 7. *Ans.* £356 11s. 10d

6. What cost 9 yards of Irish linen, at 7s. 9d. 2 qrs. per yard? *Ans.* £3 10s. 1½d.

7. What cost 11 pairs of shoes, at 10s. 6d. per pair?
Ans. £5 15s. 6d.

8. What is the weight of 9 dollars, the weight of one dollar being 17 pwt. 8gr.? *Ans.* 7oz. 16pwt.

9. What is the whole weight of 12 hogsheads of sugar, each weighing 7cwt. 3qr. 16lb.? *Ans.* 94cwt. 2qr. 24lb.

10. Multiply 3℔ 1℥ 6ʒ 2℈ 14gr. by 6.
Ans. 18℔ 11℥ 1ʒ 1℈ 4gr.

11. How many yards of cloth are in 7 pieces, each containing 29 yards 2 quarters 3 nails? *Ans.* 207yd. 3qr. 1na.

12. How many bushels of corn are in 9 bags, containing each 3 bushels 1 peck 5 quarts? *Ans.* 30bu. 2pk. 5qts.

13. How much wine is there in six casks, each containing 49 gallons 2 quarts 1 pint? *Ans.* 297gal. 3qt.

14. Multiply 6 leagues 2 miles 5 furlongs 26 poles by 8.
Ans. 55le. 5fur. 8rd.

15. How many acres in 5 fields, each containing 4 acres 1 rood 36 rods? *Ans.* 22 acres 1 rood 20 rod.

16. How much wood is there in 11 piles, each containing 2 cords 32 feet 136 inches? *Ans.* 24 cords 96ft. 1496in.

17. Multiply 12 years 6 months 3 weeks 5 days by 10.
Ans. 125yr. 9mo. 1w. 1da.

II. *When the Multiplier is a Composite Number, and greater than* 12.

RULE.

Multiply the multiplicand first by one of the component parts, and that product by another, and so on, and the last product will be the answer.

EXAMPLES.

1. What is the value of 24 yards of cloth at 6 shillings 10 pence per yard?

Operation.

	£	s.	d.	
	0	6	10	=price of 1 yard.
			6	=one of the factors.
	2	1	0	=price of 6 yards
			4	=the other factor.
Ans.	8	4	0	=price of 24 yards.

Illustration. 6 × 4 = 24, therefore we multiply the price of 1 yard first by one factor, and that product by the other, which gives the price of 24 yards.

2. What cost 54 yards of cloth at £1 3s. 6½d. a yard? *Ans.* £63 11s. 3d.

3. Multiply 5cwt. 3qrs. 15lb. by 56. *Ans.* 329cwt. 2qrs.

4. How many yards of broadcloth are there in 35 pieces, each containing 36yds. 2qr. 1na.? *Ans.* 1279yds. 2qr. 3na.

5. How much cider is there in 45 barrels, each containing 31gal. 2qts. 1pt.? *Ans.* 1423gal. 1pt.

6. Multiply 2° 58′ 10″ by 56. *Ans.* 5S. 16° 17′ 20″.

7 Multiply 24m. 3fur. 8rd. by 72. *Ans.* 1756m. 6fur. 16rd.

8. How many days in 25 years, the year containing 365d. 5h. 48m. 56s.? *Ans.* 9131ds. 1h. 23m. 20s.

9. How much wood is there in 32 loads, containing each 1 cord 16 feet 144 inches? *Ans.* 36 cords 2 feet 1152 inches.

III. *When the Multiplier is not a Composite Number.*

RULE.

Multiply by those factors whose product will come the nearest to it, then multiply the upper line by what remained, which added to the last product gives the answer.

EXAMPLES.

1. What will 47 yards of cloth come to at 14s. 6d. per yard?

	£	s.	d.	
	0	14	6	= price of 1 yard,
			9	
	6	10	6	= price of 9 yards.
			5	
	32	12	6	= price of 45 yards.
	1	9	0	= price of 2 yards.
Ans.	34	1	6	= price of 47 yards.

Illustration.—No two factors will exactly produce 47; but we multiply the price of 1 yard by 9, which gives the price of 9 yards; then multiply the price of 9 yards by 5, which gives the price of 45 yards; (9 × 5 = 45;) we then

multiply the price of 1 yard by 2 and add the product to the price of 45 yards, and the sum is equal to the price of 47 yards.

2. What will 23 yards of linen come to at 3s. 6½d. per yard? *Ans.* £4 1s. 5½d.

3. What quantity of hay is there in 17 loads, containing each 19cwt. 3qrs. 25lb.? *Ans.* 339cwt. 2qrs. 5lb.

4. How many yards of cloth are there in 29 pieces, each containing 41 yards 2qrs.? *Ans.* 1203½yds.

5. How far will a vessel sail in 34 days, at the rate of 35le. 1m. 6fur. a day? *Ans.* 1209le. 2m. 4fur.

EXAMPLES FOR EXERCISE.

1. What is the weight of 7hhds. of sugar, each weighing 7cwt. 3qrs. 19lb.? *Ans.* 55cwt. 1qr. 21lb.

2. What is the weight of 6 chests of tea, each weighing 3cwt. 2qrs. 9lb.? *Ans.* 21cwt. 1qr. 26lb.

3. In 35 pieces of cloth, each measuring 27¾ yards, how many yards? *Ans.* 971yds. 1qr.

4. How much brandy in 9 casks, containing each 45 gallons 3qts. 1pt.? *Ans.* 412gals. 3qts. 1pt.

5. How many bushels of corn in 15 barrels, each containing 3bu. 1 peck 4qts.? *Ans.* 50bu. 2pk. 4qts.

6. How far will a man travel in 18 days, at the rate of 35m. 6fur. 25 rods a day? *Ans.* 644m. 7fur. 10rds.

7. How much land is contained in 9 fields, each measuring 12 acres 2 roods 25 po.? *Ans.* 113A. 3r. 25po.

8. In 8 parcels of wood, containing each 5 cords 76 feet, how many cords? *Ans.* 44 cord 96 feet.

9. The earth performs one revolution round the sun in 365 days 5 hours 48m. 56sec. How long will it take to make 8 revolutions? *Ans.* 2921d. 22hrs. 31m. 28sec.

COMPOUND DIVISION

Teaches to find how often one given number is contained in another, of different denominations.

I. When the divisor does not exceed 12.

RULE.

Begin with the highest denomination, and divide as in Simple Division; and if there be a remainder, find how

many of the next lower denomination it is equal to, which add to the next denomination; then divide again, and carry the remainder, if any, as before: thus proceed till the whole is finished.

Proof—the same as Simple Division.

EXAMPLES.

1. If £2047 13s. 9d. be divided equally among 6 men, how much will each man receive?

Operation.

	£	s.	d.	qrs.
6)	2047	13	9	0
	341	5	7	2

Illustration.—We divide the pounds as a whole number, by 6, and the remainder, 1 pound, is equal to 20s. which we add to the next denomination, 13s. makes 33s. which divided by 6, the quotient is 5s. and the remainder 3s. We then multiply the 3s. by 12, the number of pence in one shilling, and add in the 9d. makes 45d.; this divided by 6, the quotient is 7d., and the remainder 3d., which we multiply by 4, makes 12qrs.; which being divided by 6, the quotient is 2qrs.

2. If 9 tons of hay cost £41 3s. 6d., what will 1 ton cost? *Ans.* £4 11s. 6d.

3. If 11 bushels of corn cost £2 9s. 6d. what is that a bushel? *Ans.* 4s. 6d.

4. Divide £144 10s. equally among 12 men. *Ans.* £12 0s. 10d.

5. If 26 bushels of corn be put into 8 barrels, how much will there be in each barrel? *Ans.* 3 bushels 1 peck.

6. If 9 pieces of broadcloth contain 272 yards 1 quarter, what does 1 piece contain? *Ans.* $30\frac{1}{4}$yds.

7. A tract of land, containing 96 acres 2 roods 16 rods, is to be divided into 7 equal lots; how much land will each lot contain? *Ans.* 13 acres 3 roods 8 rods.

II. If the divisor exceed 12, and be a composite number, divide first by one of the component parts, or factors, and that quotient by another, &c., and the last quotient will be the answer.

8. Divide £39 13s. 4d. by 28.

Operation.		
£	s.	d.
7)39	13	4
4)5	13	4
Ans. 1	8	4

28 being a composite number, and its component parts, or factors, 7 and 4, we first divide the 39 pounds, 13 shillings 4 pence by one of the factors, and that quotient by the other factor, which gives the true quotient.

9. If £269 12s. 4d. be divided equally among 56 men, how much will each man have? *Ans.* £4 16s. 3½d.

10. If 27 days' labour cost £10 2s. 6d, what will one day's labour come to? *Ans.* 7s. 6d.

11. Divide 59cwt. 2qrs. 14lb. by 18. *Ans.* 3cwt. 1qr. 7lb.

12. Divide 145yds. 2qrs. by 32. *Ans.* 4yds. 2qrs.+

13. Divide 94 bushels 2 pks. by 56. *Ans.* 1bu. 2pks. 6qt.

14. Divide 232 gallons, by 64. *Ans.* 3gals. 2qt. 1pt.

15. Divide 149A. 1 rood 24rds. by 96.
Ans. 1A. 2roods 9rd.

16 Divide 356yrs. 9mo. 1 week 5 days, by 108.
Ans. 3yrs. 3mo. 2 weeks 4da.

III. When the divisor exceeds 12, and is not a composite number, divide by the whole divisor at once, after the manner of Long Division.

17. Divide £176 9 shillings 6 pence, by 26.

```
     £    s. d. £.
26)176  9  6(6
   156
   ---
    20
    20
   ------- s.
26)409(15
   26
   ---
   149
   130
   ---
    19
    12
   ---- d.
26)234(9
   234
   ---
     0
```

1st. We divide the pounds, and the quotient is £6, and the remainder £20. This remainder we now multiply by 20, the number of shillings in 1 pound, and to the product add the 9 shillings in the dividend, which makes 409 shillings; this sum we divide by 26, as before, and the quotient is 15 shillings, and the remainder is 19 shillings. We then multiply the 19 shillings by 12, the number of pence in one shilling, and to the product add the 6 pence in the dividend, making 234 pence: this sum, divided by 26, as before, the quotient is 9d. We now have for the quotient £6 15s. 9d.

18. If 31 yards of cloth cost £113 13s. 4d. how much is that a yard? *Ans.* £3 13s. 4d.

19. Divide 201cwt. 3qrs. 21lb. by 41.
Ans. 4cwt. 3qr. 19lb 10oz.

20. Divide 861 yards 1qr. by 52. *Ans.* 16yds. 2qr. 1na.

21. Divide 7248 gallons, by 128. *Ans.* 56gals. 2qt. 1pt.

22. Divide 6307 miles 4fur. by 96. *Ans.* 65m. 5fur. 25rd.

23. Divide 310a. 3 roods 5rd. by 85. *Ans.* 3a. 2r. 25rd.

24. Divide 754 years 3 months 1 week, by 91.
Ans. 8yrs 3mo. 1w. 6d.

EXAMPLES FOR EXERCISE.

1. If 9 tons of hay cost £40 14s. 6d., what will 1 ton cost? *Ans.* £4 10s. 6d.

2. Divide 98lb. 2oz. 19pwt. 5grs. of silver into 7 equal parts. *Ans.* 14lb. 0oz. 8pwt. 11gr.

3. Divide 6 Ton 11cwt. 3qrs. 16lb. by 8.
Ans. 16cwt. 1qr. 26lb. 8oz.

4. If 97cwt. 2qrs. 14lbs. of sugar be contained equally in 11 hogsheads, how much sugar in 1 hogshead?
Ans. 8cwt. 3qrs. 14lb.

5. If 1113 gallons 3 quarts of brandy be put equally into 18 casks, how much will each contain?
Ans. 61 gals. 3qts. 1 pint.

6. Divide 95 gallons 3 quarts 1 pint by 25.
Ans. 3 gals. 3qts. 0pt. 2gi.+

7. Divide 94 bushels 4 quarts of corn equally among 15 men, and how much will each man have?
Ans. 6bu. 1 peck 0qt. 1pt.+

8. If a man's yearly income be £38 14s., what is that a calendar month? *Ans.* £3 4s. 6d.

9. If 88 acres 1 rood 10 rods of land be divided into 18 equal lots, how much land will each lot contain?
Ans. 4 acres 3 roods 25rd.

10. Divide 93 E.E. 4 quarters 2 nails by 7.
Ans. 13 E.E. 3qrs. 2 nails.

11. Divide 144 yards 1 quarter 2 nails by 11.
Ans. 13 yards 0qr. 2 nails

12. Divide 36 leagues 2 miles 6 furlongs 20 poles, by 15.
Ans. 2le. 1 mile 3fur. 4rd.

13. Divide 18 gallons of Ale equally among 144 men.
Ans. 1 pint each.

14. Bought 2 dozen silver spoons, which together weighed 4 pounds 3 ounces 13 pwt. What was the weight of each spoon? *Ans.* 2oz. 3pwt. 1 grain.

Questions.

What is Compound Multiplication?
1. When the multiplier does not exceed 12, how do you proceed?
2. When the multiplier is a composite number, and greater than 12, how do you proceed?
3. When the multiplier is not a composite number, how do you proceed?

What does Compound Division teach?
1. When the divisor does not exceed 12, how do you proceed?
2. If the divisor exceed 12, and be a composite number, how do you proceed?
3. When the divisor exceeds 12, and is not a composite number, how do you proceed?

REDUCTION.

Reduction teaches to bring numbers from one name, or denomination, to another, without altering their value.

Reduction is of two kinds, Descending and Ascending.

Descending is when higher denominations are brought into lower, as days into hours, &c., and is performed by multiplication.

Ascending is when lower denominations are brought into higher, as hours into days, &c., and is performed by division.

RULE.

I. *To reduce high denominations to lower.*—Multiply the highest denomination given by so many of the next less as make one of that greater, and to the product add the next lower denomination, (if any;) thus proceed with each succeeding denomination, until you have brought it to the denomination required.

II. *To reduce low denominations to higher.*—Divide the lowest denomination given by that number of the same denomination which it takes to make one of the next higher; thus proceed with each succeeding denomination, until you have brought it to the denomination required.

Proof.—Reduction Ascending and Descending alternately prove each other.

I. *Sterling Money.*

EXAMPLES.

1 In £14 18s. 4d. 2qrs., how many farthings?

Operation.

```
 £   s.  d. qrs.
14  18   4  2
20
——
298 s.
 12
————
3580 d.
   4
————
Ans. 14322 qrs.
```

Illustration.—We multiply the £14 by 20, the number of shillings in £1, adding in the 18s. which make 298s. Then as 1s. is equal to 12d. we multiply the 298s. by 12, adding in the 4d., which makes 3580 pence. Then as 1d. is equal to 4 farthings we multiply the 3580 pence by 4, adding in the 2 farthings, and the answer is 14322 farthings.

2. In 14322 farthings, how many pounds?

Operation.

```
                 qrs.
Fat. in 1d.  4)14322
             ——————
             12)3580--2qrs.
             ——————
             2'0)29'8--4d.
             ——————
                 £14--18s.
Ans. £14 18s. 4d. 2qrs.
```

We divide the farthings by 4, because 4 farthings are eqal to 1 penny, and the quotient is 3580 pence, and the remainder 2qrs.—Then because 12d. make 1 shilling, we divide the 3580 pence by 12, and the quotient is 298 shillings, and the remainder 4 pence. Then as 20 shillings make 1 pound, we divide the 298 shilling by 20, and the quotient is £14, and the remainder 18s.; and the last quotient, with the several remainders, constitutes the answer.

Note. It will be seen that this Example proves the preceding one: thus the 2d example proves the 1st, the 4th proves the 3d, and the 6th proves the 5th, and so on.

3. Reduce £94 5s. 6d. to pence.
4 Reduce 22626 penee to pounds.
5. In £256 14s. 6d. 3qrs., how many farthings?
6. In 246459qrs., how many pounds?
7. In $148, at 6s. each, how many pence?
8. In 10656d., how many dollars, at 6s. each?
9. In £125, how many half-pence?
10. In 600000 half-pence, how many pounds?

11. Reduce 35 guineas, at 28s., to pence.
12. Reduce 11760 pence to guineas, at 28s.

II. *Troy Weight.*

1. In 1lb. 11oz. 15pwt. of silver, how many grains?

```
lb. oz. pwt.
 1  11  15
 12 ounces in 1lb.
 23
 20 pwts. in 1oz.
475
 24 grs. in 1pwt.
1900
950
Ans. 11400grs.
```

2. In 11400grs. of silver, how many pounds?

```
       grs.
   6)11400
    4)1900
  2'0)47'5
     12)23–15pwt.
Answer  1 11 15
       lb. oz. pwt.
```

Note. In dividing by 24, we use the factors 6 and 4.

3. In 24lbs. 9oz. of silver, how many grains?
4. In 142560grs. of silver, how many pounds?
5. How many grains of gold are there in 16lb. 10oz. 18pwt. 5 grains?
6. In 97397grs. of gold, how many pounds?

III. *Avoirdupoise Weight.*

1. In 2T. 12cwt. 3qrs. 6lb. how many pounds?

```
T. cwt. qrs. lb.
 2  12   3   6
20 cwt. in 1 ton.
52
 4 qrs. in 1 cwt.
211
 28 lbs. in 1 qr.
1694
422
5914lbs. Ans.
```

2. Reduce 5914lbs. to tons.

```
  7)5914
   4)844+6lb.
   4)211
 2'0)5'2+3qrs.
   T. 2+12cwt.
```

Ans. 2T. 12cwt. 3qrs. 6lb.

Note. As 28lbs. are equal to 1qr., we divide the lbs. by the factors 7 and 4.

3. Bring 19lb. 11oz. 13d. into drams.
4. Bring 5053 drams into pounds.
5. In 1 ton, how many drams?
6. In 573440 drams, how many tons?

7. In 85cwt. 1qr. 15lb., how many pounds?

8. In 9563 pounds, how many cwt.?

9. In 11hhds. of sugar, each weighing 9cwt. 3qrs. 25lbs., how many pounds?

10. In 12287 pounds of sugar, how many hogsheads, each 9cwt. 3qrs. 25lbs.

11. In 156 boxes of raisins, each containing 125 pounds, how many cwt.?

12. In 174cwt. 12lb. of raisins, how many boxes of 125 pounds each?

IV. *Apothecaries Weight.*

1. Reduce 9℔ 8℥ 1ʒ to drams.

℔ ℥ ʒ
9 8 1
12
116
8
Ans. 929 drams.

2. Bring 929 drams into pounds.

8)929
12)116+1
9lb. 8oz. 1dr.

3. In 9℔ 8℥ 1ʒ 2℈ 19grs., how many grains?

4. Change 55799 grains to pounds.

V. *Cloth Measure.*

1. In 127 yards, how many quarters and nails?

127
4
508 quarters.
4
2032 nails.

2. In 2032na., how many quarters and yards?

4)2032
4)508 quarters.
127 yards.

3. In 573yds. 1qr. 1na., how many nails?

4. In 9173 nails, how many yards?

5. In 185 Ells Flemish, how many nails?

6. In 2220 nails, how many Ells Flemish?

7. In 156 Ells English, how many Ells French?

Direction.—First reduce the Ells English to quarters, then reduce the quarters to Ells French. Thus, 156 E. E. =780qrs. then 780qrs.÷6 equal to the answer.

8. Reduce 130 Ells French to Ells English.

9. In 35 pieces of cloth, each 28 yards 3 quarters, how many nails?

10. If 16100 nails of cloth be contained in 35 pieces of cloth, how many yards in each piece?

VI. *Dry Measure.*

1. In 45bu. 3pks. 5qts., how many quarts?

```
bu. pk. qt.
45  3  5
    4
  ----
  183
    8
  ----
Ans. 1469 quarts.
```

2. In 1469qts., how many bushels?

```
8)1469
  ----
4)183+5qts
  ----
bu. 45+3pks.
Ans. 45bu. 3pks. 5qts.
```

3. In 146bu., how many pecks, quarts and pints?

4. In 9344 pints, how many bushels?

5. In 35 barrels of grain, each containing 3bu. 2pks., how many pints?

6. If 7840 pints of grain be contained equally in 35 barrels, how many bushels in each barrel?

VII. *Wine Measure.*

1. In 8 tuns of wine, how many hhds., gals. and qts.?

```
   8
   4
  --
  32 hhds.
  63
  --
  96
 192
 ----
2016 gallons.
   4
 ----
8064 quarts.
```

2. In 8064 quarts of wine, how many tuns?

```
4)8064
  ----
63)2016(32
   189
   ---        4)32
    126        --
    126   Ans. 8 tuns.
    ---
      0
```

3. In 27hhds. 18gals. 2qts., how many pints?

4. In 13756 pints of wine, how many hogsheads?

5. In 25 barrels of cider, how many pints?

6. In 6300 pints of cider, how many barrels?

(*Direction.*—First reduce 6300 pints to quarts for a divi-

dend ; then reduce one barrel to quarts for a divisor. Divide, and the quotient is the answer sought.)

7. How many bottels, containing 6 gills apiece, can be filled with 3 barrels of cider ?

8. In 504 bottels of cider, each containing 6 gills, how many barrels ?

VIII. *Long Measure.*

1. In 41 miles, how many furlongs and poles ?

$$\begin{array}{r} 41 \\ 8 \\ \hline 328 \text{ furlongs.} \\ 40 \\ \hline 13120 \text{ poles.} \end{array}$$

2. In 13120 poles, how many miles ?

$$\begin{array}{r} 4'0)1312'0 \\ \hline 8)328 \\ \hline \end{array}$$

Ans. 41 miles.

3. In 46 degrees, how many rods ?

$$46 \times 69\tfrac{1}{2} \times 8 \times 40 = 1023040.$$

4. In 1023040 rods, how many degrees ?

$$1023040 \div 40 = 25576 \div 8 = 3197 \div 69\tfrac{1}{2} = 46.$$

Note. When the divisor is a mixed number, that is, a whole number joined with a fraction, $\frac{1}{2}$, $\frac{1}{4}$, &c., we must reduce the divisor to halves, or fourths, &c., and reduce the dividend to the same ; then dividing gives the true quotient. Thus, in this example, the rods being divided by 40, and that quotient by 8, we hav e 3197 miles ; then $69\frac{1}{2}$ miles = 139 half-miles, the divisor ; and 3197 miles = 6394 half-miles, the dividend.

5. In 248 miles, how many inches ?
6. In 15713280 inches, how many miles ?
7. In 120 yards 2 feet 8 inches, how many inches ?
8. Reduce 4352 inches to yards.
9. How many barley-corns will reach round the globe, it being 360 degrees ?
10. Reduce 4755801600 barley corns to degrees ?

IX. *Land or Square Measure.*

1. In 121a. 3 roods and 25rd., how many square rods ?
2. In 19595 rods, how many acres ?
3 In 256 acres, how many square feet ?

$$256 \times 4 \times 40 \times 272\tfrac{1}{4} = 11151360 \textit{ Ans}$$

4. In 11151360 square feet, how many acres?
$272\frac{1}{4}$=1089 fourths, the divisor; then 11151360= 44605440 fourths, the dividend, the quotient is 40960 square rods; then 40960 square rods = 256 acres, the answer.
5. Reduce 1 square mile, or 640 sq. acres to rods.
6. Reduce 102400 square rods to acres.

X. *Solid Measure.*

1. In 25 tons of hewn timber, how many solid inches?
2. In 2160000 solid in., how many tons of hewn timber?
3. In 25 cords of wood, how many solid inches?
4. Reduce 5529600 inches to cords.
5. In 98 tons of round timber, how many solid feet?
6. In 3920 feet of round timber, how many tons?

XI *Time.*

1. In 18d. 5h. 41m. 56s., how many seconds?
2. In 1573716 sec., how many days?
3. In 41 weeks, how many minutes?
4. Reduce 413280 minutes to weeks.
5. In 33h. 5m. 12s., how many seconds?
6. Reduce 119112s. to hours.

XII. *Circular Motion.*

1. In 6 signs 13deg. 45m. 21sec., how many seconds?
2. Reduce 697521 seconds to signs.
3. In 45° 56′ 30″, how many seconds?
4. Reduce 165390″ to degrees.

Questions.

What does Reduction teach?
Of how many kinds is Reduction?
What is Reduction Descending?
What is Reduction Ascending?
How are higher denominations reduced to lower?
How are lower denominations reduced to higher?
By what would you multiply to reduce cwts. to lbs.?—days to hours? &c.
By what would you divide to reduce lbs. to cwts.? &c.

EXAMPLES FOR EXERCISE.

1. In 20144 farthings, how many English guineas, at 28 shillings? *Ans.* 14 guineas, 27s. 8d.

2. A man borrowed 10 English guineas, and 24 English crowns, at 6s. 8d.; how many dollars at 6s., will pay the debt? *Ans.* $73\frac{1}{3}$.

3. In £48, how many shillings, nine-pences, six-pences, four-pences, and pence, and an equal number of each?

Direction.—First reduce £48 to pence for a dividend, then add together 12d., (1s.) 9d., 6d., 4d., and 1d., for a divisor. Divide, and the quotient will be the number of each?
Ans. 360.

4. A goldsmith received three ingots of silver, each weighing 27oz., which he was directed to make into spoons of 2oz., cups of 5oz., salts of 1oz., and snuff-boxes of 2oz., and to have an equal number of each. What was the number? *Ans.* 8 of each, and 1oz. over.

5. How many rings, each weighing 5pwt. 7grs., may be made of 3lb. 5oz. 16pwt. 2grs. of gold. *Ans.* 158.

6. What will 5cwt. 2qrs. 18lbs. of sugar come to, at 12 cents per pound? *Ans.* $76,08

7. What will 3 tons of iron come to, at 6 cts. per pound?
Ans. $403,20.

8. A merchant sold 12hhds. of brandy, at $16\frac{2}{3}$ cents a pint; what did it amount to? *Ans.* $1008.

9. In 12 bales of cloth, each 20 pieces, and each piece 25 Eng. Ells, how many yards? *Ans.* 7500 yards.

10. At 20 cents a nail, what is the value of 3 pieces of broadcloth, each 25 yards? *Ans.* $240.

11. How many times will a carriage wheel, 15ft. 9in. in circumference, turn round in going from Baltimore to Washington, it being 38 miles? *Ans.* 12739+times.

12. The forward wheels of a carriage are 12ft. 6in. in circumference, and the hind wheels 15 feet 9 inches; in running from New York to Boston, how many more times will the forward wheels turn round than the hind wheels, it being 223 miles? *Ans.* 19437.

13. One field contains 7 acres 2qrs., another 10 acres, a third 4 acres 1qr. 30rd., which are to be divided into 15 shares; how many rods into each share? *Ans.* 234.

14. In 29 acres 1 rood of land, how many shares of 76 rods each? *Ans.* 61 shares, and 44rds. over.

15. Admit a ship's cargo from Bordeaux to be 250 pipes, 130 hogsheads and 150 barrels: how many gallons in all, and allowing every pint to be a pound, what was the burden of the ship? *Ans.* 44415gals., and the ship's burden was
158 tons 12cwt. 2qrs.

16. How many days from the birth of Christ to Christmas, 1834, allowing the year to contain 365 days 6 hours, or $365\frac{1}{4}$ days? *Ans.* 669868d. $\frac{2}{4}=\frac{1}{2}=12$h.

17. From March 2d to November 19th following, inclusive how many days? *Ans.* 262.

18. Suppose a rail-way car to run a mile in $3\frac{1}{2}$ minutes, how long would it be at that rate in running 1280 miles? *Ans.* 3d. 2h. 40m.

19. How many strokes does a regular clock strike in 365 days, or a year? *Ans.* 56940.

20. How long would it take to count the national debt of England, which is not less than $1900,000,000, at the rate of 50 dollars a minute, reckoning, without intermission, 12 hours a day, and 365 days to the year? *Ans.* 144 years 217d. 9h. 20m.

21. There is a certain piece of land 4 rods square; how many rods does it contain? *Ans.* 16 sq. rods.

4 rods.

4 rods.

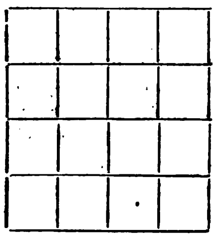

Illustration.—A piece of land 1 rod long and 1 rod wide, contains just $1\times1=1$ square rod. A piece 2 rods long and 1 rod wide contains $2\times1=2$ sq. rods, &c.; and a piece 2 rods long and 2 rods wide contains $2\times2=4$sq. rods. So a piece of land 4 rods long and 4 rods wide contains just $4\times4=16$ square rods. *See the annexed figure.*

Therefore, to find the number of square feet, rods, acres, miles, &c., in any figure that has four right angles, (corners,) multiply the length and breadth together, and the product will be the answer.

22. How many square feet in a floor that is 14ft. long and 12 feet wide? *Ans.* $14\times12=168$.

23. How many acres in a piece of land 40 rods long and 20 rods wide? *Ans.* 800 rods=5 acres.

24. How many square feet in a floor that is 4 yards long and 4 yards wide, or 4 yards square? *Ans.* 144 sq. ft.

Note.—From the foregoing it will be seen that a square foot is a space a foot long and a foot wide, but of no thickness A solid or cubic foot, is a foot long, a foot wide and

a foot thick. Thus, a block 2 feet long, 2 feet wide and 2 feet thick, contains $2\times2\times2=8$ cubic feet, &c.

25. How many solid feet in a pile of wood 8 feet long, 4 feet wide and 4 feet high? *Ans.* $8\times4\times4=128$.

26. How many cords of wood in a pile 8 feet long, 8 feet wide and 8 feet high? *Ans.* 4 cords.

27. How many cords of bark in a pile 9 feet long, 8 feet wide and 4 feet high?—Thus, $9\times8\times4=288$ feet, and 288 solid feet=2 cords 32 feet. *Ans.*

28. How many cubic inches are contained in a room 4 yards long, 4 yards wide, and 3 yards high? *Ans.* 2239488.

29. How much wood is contained in a load 8 feet long, 4 feet 6 inches high, and 3 feet 9 inches wide?

Ans. 1 cord 7 feet.

FRACTIONS.

I. Fractions or broken numbers are expressions for any part of a unit or whole number.

All fractions arise from the division of an integer or a unit into parts; thus, when a whole thing is divided into 2 equal parts, these parts are halves; when into 3 equal parts, they are thirds; when into 4 equal parts, they are fourths; when into 5 equal parts, they are fifths; when into 6 equal parts, they are sixths; when into 7 equal parts, they are sevenths; when into 8 equal parts, they are eighths; when divided into 9 equal parts, they are ninths, and so on: that is, the fraction takes its name or denomination from the number of parts into which the unit is divided. Thus, if the unit be divided into 15 equal parts, they are fifteenths; if into 20 equal parts, they are twentieths, &c.

Fractions are of two kinds, Vulgar, and Decimal.

II. A Vulgar Fraction is represented by two numbers placed one above the other, with a line drawn between them.

Thus, $\frac{1}{2}=\frac{\text{one}}{\text{half.}}$ $\frac{3}{4}=\frac{\text{three}}{\text{fourths.}}$ $\frac{5}{6}=\frac{\text{five}}{\text{sixths.}}$ $\frac{3}{8}=\frac{\text{three}}{\text{eighths.}}$ $\frac{11}{12}=\frac{\text{eleven}}{\text{twelfths.}}$

III. The number below the line is called the denominator, because it denominates or shows how many equal parts the integer or whole is divided into.

IV. The number above the line is called the numerator, because it numerates or shows how many of those equal parts are expressed by the fraction.

In division, the denominator is the divisor; and the numerator the dividend, thus:

$$\frac{\text{Dividend}=5}{\text{Divisor}=8}, \text{ shows that } 5\div 8=\frac{5}{8}.$$

1. If we divide an apple into 2 equal parts, what part of the whole apple will one of those parts be? If an apple be cut into 4 equal parts, what will one of those parts be? What part of an apple will 2 of those parts be?—will 3 be?

2. How many halves are equal to the whole of any thing? how many thirds? how many fourths? how many fifths? how many sixths? how many sevenths? how many eighths? how many ninths? how many twelfths? &c.

3. If 4 apples be equally divided among 6 boys, what part of 1 apple is each boy's share?

1 apple among 6 boys would be $\frac{1}{6}$ of an apple apiece; and 4 apples would be 4 times as much, or $\frac{4}{6}$, the answer.

4. What part of 5 dollars is 1 dollar? *Ans.* $\frac{1}{5}$.

5. What part of 8 dollars is 5 dollars? *Ans.* $\frac{5}{8}$.

6. If you cut an orange into 8 equal parts, what part of the whole orange will 1 of those parts be?

7. If I divide an orange into 8 equal parts, and give 5 of those parts to Henry, and the other three to Eliza, what part of whole orange will each have?

How are five eighths expressed?

How are three eighths expressed?

Which fraction is larger, $\frac{5}{8}$ or $\frac{3}{8}$?

V. Fractions are either proper, improper, single, compound, or mixed.

1. A simple or proper fraction is when the numerator is less than the denominator, and the fraction is less than a whole thing or unit; as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{4}$, $\frac{11}{12}$, &c.

2. An improper fraction is when the numerator is equal to, or greater than the denominator, and the fraction is equal to, or greater than a unit or whole 1; thus, $\frac{3}{3}$, $\frac{4}{2}$, $\frac{7}{5}$, $\frac{12}{4}$.

Obs. 3 thirds of any thing are equal to the whole, and 4 halves, 7 fifths, and 12 fourths, are each greater than unity or 1; consequently these are called improper fractions.

3. A compound fraction is the fraction of a fraction, coupled by the word, of, thus; $\frac{1}{2}$ of $\frac{1}{4}$, $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{8}$.

4. A mixed number is composed of a whole number and a fraction joined together, thus; $8\frac{1}{2}$, $16\frac{2}{3}$, $19\frac{3}{4}$, &c.

A whole number may be expressed as a fraction by drawing a line under it and putting 1 for a denominator; thus, $6=\frac{6}{1}$, and $12=\frac{12}{1}$, &c.

PROBLEM I.

To reduce Fractions to their lowest terms.

The numerator and denominator together are called terms of a fraction; and may often be changed without altering the value of a fraction; thus, take an orange, or any thing, and divide it into 2 equal parts, and 1 of these parts will be $\frac{1}{2}$ of the orange: again, if we divide it into 4 equal parts, it is evident that 2 of those parts ($\frac{2}{4}$) will be just $\frac{1}{2}$ of the orange: and if we divide it into 8 equal parts, 4 of these parts ($\frac{4}{8}$) will be just equal to $\frac{1}{2}$ of the orange: and the fractions $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$, are all equal in value, but expressed in different terms. Hence the terms of fractions may be changed without altering the value of the fraction; for if we multiply both the terms of the fraction $\frac{1}{2}$ by 2 it becomes $\frac{2}{4}$, which is equal to $\frac{1}{2}$: again, if we divide the terms $\frac{2}{4}$ by 2, the fraction will be $\frac{1}{2}$, which is expressed in its lowest terms possible.

RULE.

1. Divide the terms of the fraction by any number that will divide them both without a remainder.

2. Divide these quotients again in the same manner, and so on, until no number greater than 1 will divide them.

EXAMPLES.

1. Reduce $\frac{135}{225}$ to its lowest terms.
   Thus $9)\frac{135}{225}=\frac{15}{25}$, and $5)\frac{15}{25}=\frac{3}{5}$, the answer.
2. Reduce $\frac{132}{264}$ to its lowest terms. *Ans.* $\frac{1}{2}$.
3. Reduce $\frac{432}{576}$ to its lowest terms. *Ans.* $\frac{3}{4}$.
4. Reduce $\frac{147}{168}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
5. Reduce $\frac{49}{343}$ to its lowest terms. *Ans.* $\frac{1}{7}$.
6. Reduce $\frac{168}{192}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
7. Reduce $\frac{77}{84}$ to its lowest terms. *Ans.* $\frac{11}{12}$.

8. Reduce $\frac{168}{252}$ to its lowest terms. *Ans.* $\frac{2}{3}$.
9. Reduce $\frac{41}{328}$ to its lowest terms. *Ans.* $\frac{1}{8}$.
10. Reduce $\frac{252}{308}$ to its lowest terms. *Ans.* $\frac{9}{11}$.
11. Reduce $\frac{2835}{3402}$ to its lowest terms. *Ans.* $\frac{5}{6}$.

PROBLEM II.

To change a Whole or Mixed Number to an Improper Fraction.

RULE.

Multiply the whole number by the denominator of the fraction and to the product add the numerator; this sum written over the denominator will form the fraction required.

EXAMPLES.

1. In $27\frac{3}{4}$ dollars how many fourths of a dollar?

Operation.

$27\frac{3}{4}$
$+4$=fourths in 1 dollar.
108=fourths in 27 dollars.
$+3$=fourths in $\frac{3}{4}$.
111=fourths=*Ans.* $\frac{111}{4}$.

\$1=4 fourths of a dollar, and 27 dollars=27 times 4 or 108 fourths, and 3 fourths added to 108 fourths make 111 fourths=$\frac{111}{4}$ the *Ans.*

2. In $36\frac{5}{8}$ dollars, how many eighths of a dollar? *Ans.* $\frac{293}{8}$.
3. Reduce 45 to ninths. *Ans.* $\frac{405}{9}$.
4. Reduce $8\frac{5}{6}$ to an improper fraction, that is, reduce it to sixths. *Ans.* $\frac{53}{6}$.
5. Reduce $33\frac{1}{3}$ to an improper fraction. *Ans.* $\frac{100}{3}$
6. Reduce 28 to a fraction having 12 for a denominator, that is, reduce it to twelfths. *Ans.* $\frac{336}{12}$.
7. Reduce $45\frac{4}{5}$ to fifths. *Ans.* $\frac{229}{5}$.
8. Reduce $61\frac{95}{140}$ to an improper fraction. *Ans.* $\frac{8635}{140}$.
9. Reduce $84\frac{9}{11}$ to an improper fraction. *Ans.* $\frac{933}{11}$.
10. What improper fraction is equal to $56\frac{19}{40}$? *Ans.* $\frac{2259}{40}$.
11. What improper fraction is equal to $148\frac{9}{10}$? *Ans.* $\frac{1489}{10}$.
12. What improper fraction is equal to $225\frac{3}{8}$? *Ans.* $\frac{1803}{8}$.

PROBLEM III.

To change an Improper Fraction to a Whole or Mixed Number.

RULE.

Divide the numerator by the denominator, and the quotient will be the value of the fraction.

EXAMPLES.

1. In $\frac{45}{6}$ of a dollar, how many dollars?

Operation.
$\$1 = 6)45$
$\overline{\quad 7\frac{3}{6}}$

$\frac{6}{6}$ of a dollar are equal to 1 dollar, and 6 is contained in 45, 7 times and $\frac{3}{6}$ of another time; therefore the answer is $7\frac{3}{6}$ dollars $= 7\frac{1}{2}$ dollars.

2. Find the value of $\frac{100}{3}$ of a cent. *Ans.* $33\frac{1}{3}$cts.
3. Find the value of $\frac{249}{5}$ of a cwt. *Ans.* $49\frac{4}{5}$cwt.
4. Reduce $\frac{53}{6}$ to a mixed number. *Ans.* $8\frac{5}{6}$.
5. Reduce $\frac{405}{9}$ to a whole number. *Ans.* 45.
6. Reduce $\frac{336}{12}$ to a whole number. *Ans.* 28.
7. Find the value of $\frac{459}{26}$. *Ans.* $17\frac{8}{26}$.
8. Find the value of $\frac{8635}{140}$. *Ans.* $61\frac{95}{140}$

PROBLEM IV.

To Multiply a Whole Number by a Fraction.

RULE.

I. Divide the whole number by the denominator of the fraction, (when it can be done without a remainder,) and multiply the quotient by the numerator; or,

II. Multiply the whole number by the numerator of the fraction and divide the product by the denominator.

EXAMPLES.

1. What is the product of 48 multiplied by $\frac{3}{4}$?

1st method.	2d method.
4)48	48
$\overline{12}$ is $\frac{1}{4}$ of 48	3
3	4) $\overline{144} = 3$ times 48
Ans. $\overline{36} = 3$ times $\frac{1}{4}$ of 48, which is $\frac{3}{4}$ of 48.	$\overline{36}$ is $\frac{1}{4}$ of 3 times 48, which is the same as $\frac{3}{4}$ of 48

By this example we see, there are two ways of multiplying a whole number by a fraction, and that both methods produce the same result. Thus, by the first method, we get $\frac{1}{4}$ of 48, and this repeated 3 times is evidently equal to $\frac{3}{4}$, for 3 times $\frac{1}{4}$ of any number is equal to $\frac{3}{4}$ of that number. By the second method, we repeat 48, 3 times, and then take $\frac{1}{4}$ of that product, which is the same as 3 times $\frac{1}{4}$ of 48.

2. At 25 dollars per acre, what is the cost of $\frac{15}{16}$ of an acre of land? *Ans.* $23\frac{7}{16}$ dolls.

3. If a ship sail 246 miles a day, how far will she sail in $\frac{7}{9}$ of a day? *Ans.* $191\frac{1}{3}$ miles.

4. How much is $\frac{5}{6}$ of \$1845,56? *Ans.* \$1537,96$\frac{2}{3}$.

5. Multiply 400 by $\frac{3}{8}$. *Ans.* 150.

6. Multiply 750 by $\frac{3}{5}$. *Ans.* 450.

7. The interest of \$750 for 1 year, is \$45; what is the interest on the same sum for 5 months, or $\frac{5}{12}$ of a year? *Ans.* \$18,75.

Note. If the multiplier of any sum be greater than a unit or 1, the multiplicand will be increased as many times as the multiplier is greater than a unit; that is, the multiplicand will be taken as many times as the multiplier contains units. But when the multiplier is a fraction or part of a unit, the product will be only a part of the multiplicand. Hence in multiplying by a proper fraction, the product is always less than the multiplicand, as will be seen by the preceding examples.

PROBLEM V.

To Multiply a Fraction by a Whole Number.

RULE.

Multiply the whole number and the numerator of the fraction together, and write the product over the denominator; and if it produce an improper fraction, change it to a whole or mixed number, by Prob. 3.

EXAMPLES.

1. If a man spend $\frac{5}{6}$ of a dollar a day, how much will he spend in 11 days?

If he spend $\frac{5}{6}$ in 1 day, he will spend 11 times $5 = \frac{55}{6}$ in 11 days, and $\frac{55}{6}$ of a dollar $= 9\frac{1}{6}$ dollars, the answer.

2. If 1 yard of cloth cost $\frac{3}{5}$ of a dollar, what will 15 yards cost? *Ans.* \$9.

3. If a bushel of oats cost $\frac{5}{12}$ of a dollar, what will 23 bushels cost? *Ans.* \$$9\frac{7}{12}$.

4. A certain lot contains $\frac{3}{4}$ of an acre of land; how much land would 37 such lots contain? *Ans.* $27\frac{3}{4}$ acres.

5. If a bushel of potatoes cost $\frac{3}{10}$ of a dollar, what will 56 bushels cost? *Ans.* \$$16\frac{4}{5}$.

Note. The process of multiplying a fraction by a whole number, may be shortened, thus: Divide the denominator of the fraction by the whole number, (when it can be done without a remainder,) and over the quotient write the numerator.

6. If a pound of sugar cost $\frac{11}{100}$ of a dollar, what will 20lb. cost.

Divide the denominator, $_{100}$, by 20, and the quotient $_5$, is a new denominator; then write the numerator over it, and it becomes $\frac{11}{5}$ of a dollar$=2\frac{1}{5}$ dollars, the answer.

7. If a pound of nails cost $\frac{6}{66}$ of a dollar, what will 11lb. cost? *Ans.* \$1

8. If a pound of butter cost $\frac{3}{20}$ of a dollar, what will 5lb. cost? *Ans.* \$$\frac{3}{4}$.

9. At $\frac{2}{33}$ of a dollar per pound, what will 11lb. raisins come to? *Ans.* \$$\frac{2}{3}$

PROBLEM VI.

To divide a Whole Number by a Fraction.

RULE.

Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.

EXAMPLES.

1. How many times is $\frac{3}{4}$ of a dollar contained in \$9?

1 dollar is $\frac{4}{4}$, and 9 dollars is 9 times as many; $9\times4=\frac{36}{4}$; and $\frac{3}{4}$ is contained in $\frac{36}{4}$ as many times as 3 is contained in 36. *Ans* 12

2. How many times is $\frac{5}{6}$ contained in 16?

Thus, 16
$\times 6=$denominator.
Numerator$=5)\overline{96}=$sixths in 16
Ans. $19\frac{1}{5}$.

3. How many times is $\frac{2}{3}$ contained in 12? or $12 \div \frac{2}{3} =$ how many? *Ans.* 18.

4. How many times $\frac{5}{9}$ can I have in 27? *Ans.* $48\frac{3}{5}$.

5. How many times is $\frac{17}{20}$ contained in 34? *Ans.* 40.

6. How many men can I divide 75 dollars among, so as to give each $\frac{3}{4}$ of a dollar? *Ans.* 100 men.

Note. It will be seen by the 6 preceding examples, that the quotient is greater than the dividend. The reason of this is as follows. If we divide a whole number, 12 for example by 2, the quotient will be 6, which is equal to half the dividend; and if we divide it by 1, the quotient will be 12, for 1 is contained in any number twice as often as 2. Again, if we divide by $\frac{1}{2}$, the quotient will be increased, for $\frac{1}{2}$ is contained in any number twice as often as 1; thus, 12 is $= 24$ halves, and $\frac{1}{2}$ is contained in $\frac{24}{2}$, 24 times. Hence when the divisor is less than a unit, it will be contained in the dividend a greater number of times. Therefore dividing a whole number by any proper fraction, the quotient will always exceed the dividend.

PROBLEM VII.

To Reduce any given Quantity to a Fraction of a higher Denomination of the same kind.

RULE.

1. Reduce the given quantity to the lowest denomination mentioned, for a numerator.

2. Reduce 1 of the higher denomination to the same name, for a denominator.

EXAMPLES.

1. What part of 5 yards is 3 yards?

Thus, 1yd. is $\frac{1}{5}$ of 5yds., and 3 yards are 3 times as much; 3 times $\frac{1}{5}$ is $\frac{3}{5}$, the answer.

2. What part of 7lb. is 4lb.? *Ans.* $\frac{4}{7}$.

3. What part of 17 cents is 9 cents? *Ans.* $\frac{9}{17}$.

4. What part of 18 dollars is 4 dollars? *Ans.* $\frac{2}{9}$.

Note. Reduce all the fractions to their lowest terms.

5. What part of £15 is £6? *Ans.* $\frac{2}{5}$.

6. What part of 25 rods is 15 rods? *Ans.* $\frac{3}{5}$.

7. What part of 63 gallons is 9 gallons? *Ans.* $\frac{1}{7}$.

8. What part of 19 acres is 5 acres? *Ans.* $\frac{5}{19}$.
9. 18 inches is what part of 56 inches? *Ans.* $\frac{9}{28}$.
10. What part of £1 is 12s. 9d. 3qrs.?

Operation.

12s. 9d. 3qrs.	£1=20s.
12	12
153	240
4	4

Numerator 615qrs. Denominator 960qrs. $=\frac{615}{960}=\frac{41}{64}$.

11. What part of a shilling is $4\frac{1}{2}$d.? *Ans.* $\frac{3}{8}$.
12. What part of a pound Troy is 7 oz. 4pwt. *Ans.* $\frac{3}{5}$.
13. What part of 1cwt. is 2qr. 16lb.? *Ans.* $\frac{9}{14}$.
14. What part of a yard is 3qr. 3na.? *Ans.* $\frac{15}{16}$.
15. What part of a hogshead is 35gal. 2qts.? *Ans.* $\frac{71}{126}$.
16. What part of a furlong is 6rd. 3yds. 2feet? *Ans.* $\frac{1}{6}$.
17. What part of an acre is 3 roods 21rds.? *Ans.* $\frac{141}{160}$.
18. Reduce 54 gallons to the fraction of a hogshead. *Ans.* $\frac{6}{7}$.
19. Reduce 6fur. 26rds. 11ft. to the fraction of a mile. *Ans.* $\frac{5}{6}$.
20. What part of 1 year is 7 weeks 1 day? *Ans.* $\frac{10}{73}$.
21. What part of a yard is 2qr. $2\frac{2}{3}$na.? *Ans.* $\frac{2}{3}$yd.

(Reduce 2qr. 2na. to nails, then reduce the nails to thirds, adding in the 2 thirds, for the numerator; then reduce 1 yard to thirds of nails, for the denominator.)

22. What part of a day is 11 hours $5\frac{6}{7}$ minutes? *Ans.* $\frac{4661}{10080}$.
23. Reduce 4 shillings 6 pence to the fraction of a dollar or 6 shillings. *Ans.* $\$\frac{3}{4}$
24. What part of the year had transpired the 26th day of October, 1836, including that day? *Ans.* $\frac{61}{81}$.
25. What part of a dollar at 8 shillings, is 2 shillings 8 pence? *Ans.* $\$\frac{1}{3}$.

PROBLEM VIII.

To find the Value of a Fraction in Whole Numbers of less Denominations.

RULE.

1. Multiply the numerator by the parts in the next lower denomination, and divide the product by the denominator.

2. Multiply the remainder, if any, by the next lower denomination, and divide by the denominator, as before; and the several quotients will be the answer.

EXAMPLES.

How much is $\frac{1}{2}$lb. avoirdupoise? How much is $\frac{1}{4}$lb.? How much is $\frac{1}{8}$lb.? How much is $\frac{3}{8}$lb.? How much is $\frac{5}{8}$lb.? How much is $\frac{7}{8}$lb.? How much is $\frac{1}{4}$ of a shilling? How much is $\frac{1}{2}$ of a shilling? How much is $\frac{3}{4}$ of a shilling? How much is $\frac{1}{3}$ of a shilling? How much is $\frac{2}{3}$? How much is $\frac{1}{6}$ of a shilling?

1. What is the value of $\frac{7}{8}$ of a pound sterling?

Operation.

```
        Numerator=7
Shillings in £1=×20
               ——— s.
 Denominator=8)140(17
               8
              ———
               60
               56
              ———
                4
Pence in 1shil.=12
                — d.
             8)48(6
               48
               —
```

£1=20s. and $\frac{7}{8}$ of £1 is same as $\frac{7}{8}$ of 20s.; and to get $\frac{7}{8}$ of 20s. we multiply the numerator of the fraction, 7, and 20, together; and the product divided by the denominator, 8, gives 17s. and $\frac{4}{8}$ of another shilling remaining; then $\frac{4}{8}$ of a shilling is $\frac{4}{8}$ of 12 pence, and to get $\frac{4}{8}$ of 12d. we multiply the numerator, 4, and 12, together, and divide by the denominator, gives 6 pence.

Ans. 17s. 6d.

2. What is the value of $\frac{11}{12}$ of a pound sterling? *Ans.* 18s. 4d.

3. What is the value of $\frac{3}{8}$ of a shilling? *Ans.* $4\frac{1}{2}$.

4 What is the value of $\frac{6}{7}$ of a shilling? *Ans.* 10 pence $1\frac{1}{7}$qrs.

5. What is the value of $\frac{24}{32}$ of a pound Troy? *Ans.* 9oz.

6. What is the value of $\frac{4}{5}$ of a pound avoirdupoise? *Ans.* 12oz. $12\frac{4}{5}$dr.

7. Reducc $\frac{7}{9}$ of a hundred weight to its proper quantity? *Ans.* 3qr. 3lb. 1oz. $12\frac{4}{9}$dr.

8. What is the value of $\frac{5}{9}$ of an Ell English? *Ans.* 2qr. $3\frac{1}{9}$na.

9. What is the value of $\frac{6}{7}$ of a yard? *Ans.* 3qr. $1\frac{5}{7}$na.

10. How much is $\frac{15}{27}$ of a hogshead of wine? *Ans.* 35gals.

11. How much is $\frac{5}{6}$ of a mile? *Ans.* 6fur. 26rd. 3yds. 2ft.

12. How much is $\frac{7}{13}$ of a day? *Ans.* 12h. 55min. $23\frac{1}{13}$sec.

13. How much is $\frac{29}{32}$ of an acre? *Ans.* 3 roods. 25 rods.

Questions

I. What are Fractions? From what do all fractions arise? Of how many kinds are fractions?

II. How is a Vulgar fraction represented?

III. What is the number below the line called? and why?

IV. What is the number above the line called? and why?

In Division, what is the denominator? and what is the numerator?

V. What is a simple or proper fraction? What is an improper fraction? What is a mixed nnmber? How may a whole number be expressed as a fraction?

Prob. I. How do you reduce fractions to their lowest terms? What are the terms of a fraction?

II. How do you change a whole or mixed number to an improper fraction?

III. How do you change an improper fraction to a whole or mixed number?

IV. How do you multiply a whole number by a fraction?

V. How do you multiply a fraction by a whole number?

VI. How do you divide a whole number by a fraction?

VII. How do you reduce a given quantity to the fraction of a greater denomination of the same kind?

VIII. How do you find the value of a fraction in whole numbers of less denominations?

DECIMAL FRACTIONS.

1. A Decimal* Fraction is that whose denominator is always 1 with a cipher, or a number of ciphers annexed to it. Thus, $\frac{5}{10}$, $\frac{6}{100}$, $\frac{56}{1000}$, &c. &c.

2. The integer is always divided into 10, 100, 1000, &c. equal parts. Therefore the denominator is always 10, 100, 1000, &c.

3. The true value of a decimal fraction is expressed by writing the numerator only with a point before it. Thus, $\frac{5}{10}$ is written, ,5 ; $\frac{25}{100}$, ,25 ; $\frac{645}{1000}$, ,645.

4. If the numerator has not so many places of figures as the denominator has ciphers, we must put as many ciphers on the left hand as will make up the defect. Thus, $\frac{6}{100}$ is written ,06 and $\frac{6}{1000}$ is written ,006, &c.

5. The point prefixed is called a separatrix.

---

* So called from the Latin word *decem*, which signifies ten.

6. Each figure takes its value by its distance from the unit's place; the first figure on the right hand of units, or the separatrix, signifies so many tenths; the second so many hundredths; the third so many thousandths, &c., thus decreasing in a tenfold proportion from the left towards the right hand.

7. Ciphers placed at the right hand of a decimal fraction do not alter its value, since every significant figure continues to possess the same place. Thus, ,5 ,50 ,500, &c. are all of the same value, and each equal to $\frac{5}{10}$ or $\frac{1}{2}$.

8. Every cipher placed at the left hand of a decimal fraction decreases its value tenfold, by removing each significant figure farther from the place of units. Thus, ,5 in the first place is 5 tenths; ,05 in the second place is 5 hundredths; ,005 in the third place is 5 thousandths, &c. *See the following*

TABLE.

	Hundreds.	Tens.	Units.	Tenths.	Hundredths.	Thousandths.	Ten thousandths.	Hundred thousandths.	Millionths.	
$\frac{6}{10}=$	-	-	-	,6	-	-	-	-	-	=6 tenths.
$\frac{5}{100}=$	-	-	-	,0	5	-	-	-	-	" 5 hundredths.
$\frac{65}{1000}=$	-	-	-	,0	6	5	-	-	-	" 65 thousandths.
$\frac{1234}{10000}=$	-	-	-	,1	2	3	4	-	-	" 1234 ten thousandths.
$\frac{45012}{100000}=$	-	-	-	,4	5	0	1	2	-	" 45012 hun. thousandths
$8\frac{5}{1000000}=$	-	-	8	,0	0	0	0	0	5	" 8 and 5 millionths.
$45\frac{65}{100}=$	-	4	5	,6	5	-	-	-	-	" 45 and 65 hundredths.
$365\frac{123456}{1000000}=$	3	6	5	,1	2	3	4	5	6	" 365 & 123456 millionths

Whole numbers. Decimal parts.

9. The magnitude of a decimal fraction depends mostly on the first, or left hand figure, which, if it be less than ,9, we may extend to an infinite number of figures, and it will not be equal to ,9. Thus, ,899999 is not equal to ,9.

10. Decimals are read in the same manner as whole num-

bers, giving the name of the lowest denomination, or right hand figure to the whole. Thus ,52 is read 52 hundredths; ,425 is read 425 thousandths, &c.

11. When whole numbers and decimals are expressed in the same number, it is called a *mixed* number.

Write, in decimals, the following mixed numbers.

1. Twenty-five and six tenths=25,6.
2. Sixteen, and seventy-five hundredths.
3. Forty-one, and one hundred and forty-five thousandths.
4. 362, and nine millionths.
5. Nineteen, and 34756 hundred thousandths.
6. 794, and twenty-five ten thousandths.

$10\frac{9}{10}$; $56\frac{84}{100}$; $41\frac{51}{1000}$; $95\frac{12}{10000}$; $145\frac{24}{1000000}$, &c.

ADDITION OF DECIMALS.

RULE.

1. Place the numbers, whether mixed or pure decimals, under each other, according to the value of their places.

2. Add them together as whole numbers, and place the separatrix exactly under the separating point above.

EXAMPLES.

1. What is the sum of 28,753; 365,41; 18,75; 145,6.

28,753 365,41 18,75 145,6 ——— 558,513	We place tenths under tenths, hundredths under hundredths, &c., according to the value of their places, and add the columns as whole numbers; and the amount is 558 units, and 513 thousandths of a unit.

Note. We may here observe, that the denominations of Federal Money correspond exactly with decimals, the dollars being units, dimes being tenths, cents hundredths, and mills thousandths of dollars, &c.

2. Add together the following sums of dollars and decimals of a dollar, viz: 13,755, 2,50, 25,3, and 41,144.

Ans. \$82,699=82dols. 69cts. 9m

3. Find the amount of 79,45dols., 36,045dols., 128,5 dols., 95,006dols., 135,25dols., and 14,146dols.

Ans. 488dols. 39cts. 7m.

4. Add together the following mixed numbers, viz: 5,98471 + 18,568 + 2,005 + 9,15 + 35,1009 + ,35762.
Ans. 71,16623.

5. What is the whole sum of 5,91 acres, 3,5 acres, 8,596 acres, ,795 acres, and 14 acres? *Ans.* 32,801 acres.

6. Required the sum of 25,164lbs., 9,56lbs., 87,31lbs., 256,25lbs., 9,18lbs., and 125,9lbs. *Ans.* 513,364lbs.

7. Add together 276, 54,321, ,65, 112, 12,5 and ,0463.
Ans. 455,5173

8. What is the sum of $5\frac{9}{10}$ ounces, $7\frac{95}{100}$ ounces, $3\frac{25}{1000}$ ounces, $15\frac{5}{100}$ ounces and $25\frac{6}{100}$ ounces? *Ans.* 56,985.

9. Find the amount of forty-five hundredths, two hundred and fifty-six thousandths, sixty-five hundredths, ten, and six hundred and forty-four thousandths. *Ans.* 12.

SUBTRACTION OF DECIMALS.

RULE.

Place the numbers according to their value, then subtract as in whole numbers, and point off the decimals as in addition of decimals.

EXAMPLES.

1. From	215,64	2. From	95,3465
Subtract	141,56875	Take	14,04
Remains	74,07125		81,3065

3. From 356 take 198,5693. *Ans.* 157,4307.
4. From $51,978 take $25,36. *Ans.* $26,618.
5. From 480, take 245,0075. *Ans.* 234,9925.
6. From 295 subtract 201,9. *Ans.* 93,1.
7. From 319, take ,9125. *Ans.* 318,0875.
8. From 35000, take ,035. *Ans.* 34999,965.
9. From 271, take 213,95. *Ans.* 57,05.
10. From 10, subtract one millionth part of a unit.
Ans. 9,999999.

MULTIPLICATION OF DECIMALS.

RULE.

Multiply as in whole numbers, and point off as many figures in the product for decimals, as there are decimal

places in both factors. If the number of figures in the product be less than the decimal places in both factors, prefix ciphers to the left to supply the defect.

EXAMPLES.

1. Multiply 75 by ,8.

Operation.
75
,8
Ans. =60,0

Multiplying by a fraction is taking a certain part of the multiplicand for the product: consequently, multiplying a whole number by a certain part of a whole number, as 8 tenths, produces a product less than the whole number: in this example, the number of decimal places in the factors being one, therefore we point off one figure in the product, and the answer is 60. Likewise multiplying one decimal fraction by another, produces a fraction smaller than either of the factors.

2 Multiply 4,25 by 3,6.

3,6
2550
1275
Ans. 15,300

In this example, the whole number of decimal places in both multiplier and multiplicand is three, therefore we point off three figures in the product.

3. Multiply 5,34 by ,008.

,008
,04272

In this example, the number of figures in the product is less than the decimal places in both factors; the defect must be supplied by prefixing a cipher; that is, placing it at the left hand.

4. Multiply 36,5 by 7,27. *Ans.* 265,355.
5. Multiply 3,92 by 196. *Ans.* 768,32.
6. Multiply 29,831 by ,952. *Ans.* 28,399112.
7. Multiply 79,347 by 23,15. *Ans.* 1836,88305.
8. Multiply ,009 by ,009. *Ans.* ,000081.
9. Multiply 25 dollars by 25 cents, and what is the product? *Ans.* \$6,25.
10. What cost 8,75 yards of cloth, at \$3,96 per yard? *Ans.* \$34,65.
11. What cost 18,75 barrels of flour, at \$6,75 per barrel? *Ans.* \$126, 56c. $2\frac{5}{10}$m.
12. What is the value of 18,25lbs. of butter, at \$,125 per pound? *Ans.* \$2 28c. $1\frac{25}{100}$m.

13. At ,03cts. profit on a dollar, what is the profit on $18,75? *Ans.* 56cts. $2\frac{5}{10}$m.

14. Multiply 135 dollars by $,06 or cents. *Ans.* $8,10.

15. Multiply $14,56 by $1,25. *Ans.* $18,20.

16. Multiply 3672 by ,85. *Ans.* 3121,2.

17. Multiply 235,45 dollars by ,007, or 7 mills.
Ans. 1dol. 64cts. $8\frac{15}{100}$m.

18. Multiply $,95 or 95cts. by $,125, or 12cts. 5m.
Ans. 11cts. $8\frac{75}{100}$m.

19. How much is ,5 of 138. *Ans.* 69.

20. How much is 6 per cent, or ,06 of $1495?
Ans. $89,70.

Note. To multiply by 10, 100, 1000, &c., remove the decimal point as many places to the right as the multiplier has ciphers.

Thus, ,365	multiplied by	10	is	3,65
	" "	100	is	36,5
	" "	1000	is	365.

DIVISION OF DECIMALS.

Division of decimals differs from the division of whole numbers only in pointing off the decimal places. We have seen, in Multiplication of decimals, that the decimal places in the product must be equal to the number of decimal places in both factors counted together.—So in division of decimals, the number of decimal places in the divisor and quotient counted together, must be equal to the number of decimal places in the dividend; because the divisor and quotient are the factors which produce the dividend.

RULE.

1. Divide as in whole numbers, and from the right hand in the quotient point off as many figures for decimals, as the decimal places in the dividend exceed those in the divisor.

2. If the places in the quotient be not so many as the rule requires, supply the deficiency by prefixing ciphers.

3. If the divisor has more decimal places than the dividend, annex as many ciphers as you please to the dividend so as to make it equal at least to the divisor. [illegible] also annex ciphers to the remainder, if any, and [illegible]

quotient to any degree of exactness; but the ciphers annexed must be counted as so many decimals of the dividend.

EXAMPLES.

1. Divide 89,756 by ,8.

,8)89,7560
112,195

We divide as in whole numbers, and there being a remainder, we annex a cipher and divide: there are now four decimal places in the dividend and one in the divisor. We therefore, by the rule, point off three figures in the quotient for decimals, which makes the number of decimal places in the divisor and quotient counted together, equal to the number of decimal places in the dividend.

2. Divide ,36792 by 4,2.

```
4,2),36792(,0876
     336
     ---
      319
      294
      ---
       252
       252
       ---
         0
```

In this example, there are five decimal places in the dividend, and only one in the divisor; therefore we must point off four figures in the quotient. Now, because there are only three figures in the quotient, we place a cipher on the left, and the decimal places in the divisor and quotient counted together, are equal to the decimal places in the dividend.

3. Divide 44,98 by 1,3. *Ans.* 34,6.
4. Divide 14, by ,7854. *Ans.* 17,825.
5. Divide 6,9564 by 856. *Ans.* ,00812 ×.
6. Divide ,009564 by ,008. *Ans.* 1,1955.
7. Divide ,07646 by 246. *Ans.* ,00031 ×.
8. Divide 16 by 248. *Ans.* ,0645 ×.
9. Divide $256,125 by 12,5. *Ans.* $20,49.
10. Divide $510, by $1256. *Ans.* $,40605 ×.
11. If 8,75 yards of cloth cost $34,65, what is that a yard? *Ans.* $3,96.
12. Bought 18,75 barrels of flour for $126,5625; how much was that a barrel? *Ans.* $6,75cts.
13. If 148,5lbs. of butter cost $18,5625, what will 1lb. cost? *Ans.* $,125=12½cts.
14. At $1,79 per barrel, how many barrels of cider can be bought for $270,29? *Ans.* 151.
15. If a bushel of wheat cost $1,87, how many bushels can be bought for $28,985? *Ans.* 15,5bu.=15½bu.

16. How many times 6,25 dollars can I have in 1235 dollars? *Ans.* 197,6.

Note.—To divide by 10, 100, 1000, &c., remove the decimal point in the dividend as many places towards the left hand as there are ciphers in the divisor.

Thus, 425,	divided by	10,	the quotient is	42,5
42,5	"	10,	"	4,25
425,4	"	100,	"	4,254
425,	"	1000,	"	,425.

REDUCTION OF DECIMALS.

PROBLEM I.

To reduce a Vulgar Fraction to its equivalent decimal.

RULE.

Annex ciphers to the numerator of the given fraction, and divide by the denominator, the quotient will be the decimal required, which must contain as many decimal places as there are ciphers annexed to the numerator. If there are not so many figures in the quotient, make up the deficiency by placing ciphers on the left.

EXAMPLES.

1. Reduce $\frac{7}{8}$ to its equivalent decimal.

Operation.
8)7,000
,875 Answer.

2. Reduce $\frac{5}{9}$ to a decimal fraction.

Operation.
9)5,00000
,55555

We might continue annexing ciphers to this remainder, and carry on the quotient still lower; but were it carried to an infinite number of figures, we should never arrive at a complete quotient.*

3. Reduce $\frac{1}{2}$ to a decimal fraction. *Ans.* ,5.
4. Reduce $\frac{3}{4}$ to a decimal. *Ans.* ,75.
5. What decimal is equal to $\frac{1}{5}$? *Ans.* ,2.
6. Reduce $\frac{11}{12}$ to a decimal. *Ans.* ,91666+
7. Reduce $\frac{3}{25}$ to a decimal. *Ans.* ,12.

* Such fractions are called circulating, or repeating decimals.

8. Reduce $\frac{2}{48}$ to a decimal. *Ans.* ,04166+.
9. Reduce $\frac{3}{9}$ to a decimal. *Ans.* ,33333+.
10. Reduce $\frac{1}{26}$ to its equal decimal. *Ans.* ,03846+.
11. Reduce $\frac{9}{1426}$ to a decimal. *Ans.* ,00631+.
12. Reduce $\frac{7}{8}$ to its equivalent decimal. *Ans.* ,875.

PROBLEM II.

To reduce quantities of different denominations to a decimal of the highest denomination.

RULE.

1. Reduce the given denominations to a Vulgar Fraction, as taught in *Problem* 7, *page* 93; then reduce the Vulgar Fraction to its equivalent decimal.

EXAMPLES.

1. Reduce 8s. 7d. 2qrs. to the decimal of a pound.

```
960)414,00000(,43125 Ans.        8s. 7d. 2qrs.
    3840                         12
    ----                         --          414
     3000                        103       £1=960 qrs.
     2880                          4
     ----                        ---
      1200                       414 qrs.
       960
      ----
      2400
      1920
      ----
       4800
       4800
       ----
```

Note. This Rule will give the true decimal; but the following Rule is much easier, and therefore the best for practice. *See this example performed by Rule* 2.

Rule 2. Write the denominations below each other, placing the lowest name at the top; then divide each denomination (beginning at the top,) by that number which makes one of the next higher, and the last quotient will be the decimal sought.

Perform the preceding example by this Rule.

```
Thus  4|2,
       |--
     12|7,5
       |---
    2'0|8,625
       |-----
       | ,43125
```

In dividing by the several divisors, we annex as many ciphers to each dividend as are necessary. Thus we annex 1 cipher to the farthings, and divide by the number of qrs. in a penny, and so on with each denomination.

2. Reduce 7s. 6d. to the decimal of a pound. *Ans.* ,375

3. Reduce 14s. 9d. 3qrs. to the decimal of a pound. *Ans.* ,740625.

4. Reduce 6d. to the decimal of a shilling. *Ans.* ,5

5. Reduce 15s. 9d. 3qrs. to the decimal of a pound. *Ans.* ,790625.

6. Reduce 5s. 3d. to the decimal of a dollar, New England currency, (=6s.) *Ans.* ,875.

7. Reduce 14s. to the decimal of a pound. *Ans.* ,7.

Note. If the shillings be an even number, half that number, with a point prefixed, is their decimal expression; but if the number be odd, annex a cipher; then half that number is the decimal required.

8. Reduce 1, 2, 3, 4, 9, 16, 17, and 19 shillings to decimals.

Thus	1,0	2	3,0	4	9,0	16	17,0	19,0
Answers	,05	,1	,15	,2	,45	,8	,85	,95

9. Reduce £18 2s. 7d. to a decimal expression. *Ans.* £18,129166+.

10. What is the decimal expression of £25 18s. 6¼d.? *Ans.* £25,926041+.

11. Reduce 9oz. 13pwt. to the decimal of a pound Troy. *Ans.* ,80416+.

12. Reduce 3qrs. 25lb. to the decimal of a cwt. Avoirdupois,

lbs. qrs.

Thus 28)25,0000(,8928+qrs.; then 4)3,8928

Answer ,9732+.

13. Reduce 3qrs. 2na. to the decimal of a yard. *Ans.* ,875.

14. Reduce 3pks. 2qts. 1pt. to the decimal of a bushel. *Ans.* ,828125.

15. Reduce 45 gals. to the decimal of a hogshead. *Ans.* ,7142+.

16. Reduce 2 feet 9 inches to the decimal of a yard. *Ans.* ,91666+.

17. Reduce 6fur. 26rd. 2yd. to the decimal of a mile. (By Rule I.) *Ans.* ,8323+.

18. Reduce 2 roods 24rds. to the decimal of an acre. *Ans.* ,65.

19. Reduce 4 months 2 weeks to the decimal of a year. *Ans.* ,375

PROBLEM III.

To find the decimal (to three places of figures,) of any number of shillings, pence and farthings, by Inspection.

RULE.

1. Write half the greatest even number of shillings for the first decimal figure; and if the shillings be odd, increase the second place, or place of hundredths, by 5.

2. Let the farthings in the given pence and farthings possess the second and third places, increasing the third place by 1 when the farthings exceed 12, and by 2 when they exceed 36.*

EXAMPLES.

1. Reduce 17s. 6d. 2qrs. to the decimal of a pound.

Thus, ,8 =half the greatest even number of shillings.
5 the s. are odd, therefore we write 5 hund'ths.
26=farthings in 6d. 2qrs.
1 we increase by 1, because the qrs. exceed 12.
£ ,877 decimal required.

2. Reduce 8s. $5\frac{3}{4}$d. to the decimal of a £. *Ans.* ,424.

3. Reduce 19s. 4d.; 5s. $6\frac{1}{2}$d.; 6s. $4\frac{3}{4}$d., and 12s. 6d. to decimals of a pound. *Ans.* ,967; ,277; ,320, and ,625.

4. Express £5 15s. $11\frac{1}{4}$d. decimally. *Ans.* £5,797.

5. Express £44 18s. $6\frac{1}{4}$d. decimally. *Ans.* £44,926

PROBLEM IV.

To find the value of a decimal in whole numbers of a lower denomination.

RULE.

1. Multiply the given decimal by the number of parts in the next less denomination, and point off as many decimal places as there are in the given decimal.

---

* The reasoning of this Rule is as follows,—shillings are 20ths of a pound. Thus 1s. $\frac{1}{20}$=£,05; and 2s. $\frac{2}{20}$=£,1; and 4s. $\frac{4}{20}$ £,2: thus $\frac{1}{2}$ the number of shillings is so many 10ths of a £ and each odd shilling is ,50 £.

Then each farthing is $\frac{1}{960}$ of a £. Had it happened that 1000 instead of 960 had made a pound, then the farthings would have been so many thousandths. But 960, increased by $\frac{1}{24}$ part of itself, is=1000. Therefore, any number of farthings, increased by their $\frac{1}{24}$ part, will be an exact decimal. Hence, when the farthings are more than 12, $\frac{1}{24}$ part is more than $\frac{1}{2}$ a farthing, and we add 1; and when they are more than 36, $\frac{1}{24}$ part is greater than $1\frac{1}{2}$, and we add 2.

2. Multiply this decimal by the next inferior denomination, and point off the decimals as before: proceed in this manner through all the parts of the integer, and the several denominations standing on the left of the decimal point, constitute the answer.

EXAMPLES.

1. Reduce ,695 of a pound to its proper value in whole numbers of a smaller denomination.

```
       ,695
         20
   ---------
s.   13,900
         12
   ---------
d.   10,800
          4
   ---------
qrs.  3,200
```

We multiply the decimal by 20, because £1=20s., and the product is 13,900s.; and because 1s.=12d. we multiply this decimal by 12, which gives 10,800d.; lastly we multiply by 4, because 1d.=4qrs.

Ans. 13s. 10d. $3\frac{2}{10}$qrs.

2. What is the value of ,95 of a pound? *Ans* 19s.
3. What is the value of ,625 of a shilling? *Ans*, $7\frac{1}{2}$d.
4. Find the value of ,640625 of a £. *Ans*. 12. $9\frac{3}{4}$d
5. Find the value of ,0356 of a pound. *Ans*. $8\frac{1}{2}$d.
6. Reduce ,857 of a shilling to pence and farthings. *Ans*. 10d. $1\frac{1}{10}$qr.
7. Reduce ,945 of a lb Troy to oz. pwts. and grs. *Ans*. 11oz. 6pwt. $19\frac{2}{10}$grs.
8. Reduce ,6725 of a cwt. to qrs, lbs. oz. &c. *Ans*. 2qrs. 19lbs. 5oz.
9. Reduce ,954 of a yard to qrs. and nails. *Ans*, 3qrs. 3+na.
10. Reduce ,725 of a hogshead to gals. qts. and pts. *Ans*. 45gals. 2qt. 1,4pt.
11. Reduce ,4021 of a mile to its proper quantity. *Ans*. 3fur. 8rds. 3yds. 2ft. 1+in.
12. What is the value of ,96875 of an acre? *Ans*. 3 roods 35 rods.
13. Reduce ,0546875 of a lb. avoirdupoise to its proper quantity. *Ans*. 14dr.
14. Change £45,940625 to its proper expression in pounds, shillings, &c. *Ans*. £45 18s. $9\frac{3}{4}$d.
15. Reduce ,569 of a year to days, hours, minutes and seconds. *Ans*. 207d. 16h. 26m. 24sec.

PROBLEM V.

To find the Value of any decimal of a pound (£) by Inspection.

RULE.

Double the first figure, or figure of tenths, for shillings; then if the second figure be 5, or more, deduct 5 from it, and reckon another shilling; then call the remaining figures in the second and third places so many farthings, subtracting 1 when they are above 12, and 2 when they are above 36.

EXAMPLES.

1. Find the value of ,785 of a pound.

Double 7, the first figure, or tenths, for s. 14s. " "

Then the second figure being more than 5, we deduct 5 from it, and add 1 to the shillings. 1s. " "

Then the remaining figures, 35, we call so many qrs.; abating 1 because they are more than 12 and less than 36, leaves 34qrs. And 34qrs.=8d. 2qrs. " 8d. 2qrs.

Ans. 15s. 8d. 2qrs.

2. Reduce ,875 of a pound to shillings, pence and farthings. *Ans.* 17s. 6d.

3. Reduce ,095 of a pound to its proper quantity. *Ans.* 1s. $10\frac{3}{4}$d.

4. Find the value of £ ,230. *Ans.* 4s. $7\frac{1}{4}$d.

Note. When the decimal has but two places of figures, annex a cipher to it, or suppose a cipher to be annexed.

5. Find the value of ,76 of a pound. *Ans.* 15s. $2\frac{1}{2}$d.

6. Find the value of ,34 of a pound. *Ans.* 6s. $9\frac{1}{2}$d.

7. Find the value of ,95 of a pound. *Ans.* 19s.

Questions.

1. What is a Decimal Fraction?
2. How is the integer divided?
3. How is the true value of a decimal fraction expressed?
4. If the numerator has not so many places as the denominator has ciphers, how do you write it?
5. By what does each figure take its value? What is the first figure on the right hand of units, or the separatrix?—the second?—the third?—the fourth? &c.
6. What effect do ciphers have when placed on the right of decimals?
7. When placed on the left, what effect do they have?
8. On what does the magnitude of a decimal fraction depend?

9. How are decimals read?

10. When whole numbers and decimals are expressed in the same number, what is it called?

11. What is the Rule for Addition of decimals?—for Subtraction?

12. What is the Rule for Multiplication of decimals?

13. What is the Rule for Division of decimals?

I. How do you reduce a vulgar fraction to its equivalent decimal?

II. How do you reduce quantities of several denominations to a decimal of the highest?

III. How do you find the decimal of any number of pounds, shillings, pence and farthings, by Inspection?

IV. How do you find the value of a decimal in whole numbers of a lower denomination?

V. How do you find the value of any decimal of a pound by Inspection?

REDUCTION OF CURRENCIES.

Formerly the pound was of the same value in Great Britain and all the American Colonies, (now States,) and the dollar reckoned at 4s. 6d.

But the Legislatures of the different States issued bills of credit, which depreciated in their value, in some States more, and in others less, which caused the currencies of the several States to differ from each other.

Thus, a dollar is reckoned

In the New England States, also Virginia, Kentucky, and Tennessee,	at 6s. called New England currency.
In New York, North Carolina and Ohio,	at 8s., called New York currency.
In N. Jersey, Pennsylvania, Delaware and Maryland,	at 7s. 6d., called Pennsylvania currency.
In South Carolina and Georgia,	at 4s. 8d., called Georgia currency.
In Canada and Nova Scotia,	at 5s., called Canada currency.

PROBLEM I.

To reduce the currencies of the several States in which a dollar is an even number of shillings, to Federal money, viz:

New England,	New York,
Virginia,	North Carolina,
Kentucky, and	and
Tennessee.	Ohio.

RULE.

1. When the sum consists of pounds only, annex a cipher to the pounds, and divide by half the number of shillings in a dollar, the quotient will be dollars, &c.

2. If the sum consists of pounds, shillings, pence, &c. reduce the pounds and shillings to shillings, and the pence and farthings to the decimal of a shilling; annex this decimal to the shillings, with a point between; then divide the whole by the number of shillings in a dollar, and the quotient will be dollars, cents, mills, &c.

EXAMPLES.

1. Reduce £348, New England currency, to Federal money.

```
    3)3480
Ans. $1160
```

Annexing a cipher to £348 reduces them to tenths of pounds = 3480 tenths. Then a dollar, New England currency, is $\frac{6}{20}$ = 3 tenths of a pound. And 3480 tenths, divided by 3 tenths, the quotient is 1160 dollars.

2. Reduce £145, New York, &c., currency, to Federal money.

```
4) 1450
   362,5
```

We annex a cipher to the pounds, as before; then a dollar in this currency is $\frac{8}{20}$ of a pound, = £ ,4. Therefore we divide by 4, and the quotient is 362 dollars; and to the remainder we may annex a cipher, and divide, which gives 50 cents. *Ans.* $362 50cts.

3. Reduce £25 6s. 8¾d., New England currency, to Federal money.

```
   £25 6s.
     20            4|3
6)506,7291        12|8,75
   84,4548+           ,7291
```

Ans. $84 45cts. $4\frac{8}{10}$m.

This sum, consisting of pounds, shillings, pence, &c., we reduce the pounds and shillings to shillings, and the pence and qrs. to the decimal of a shilling and divide by the number of shillings in a dollar.

4. Reduce £56 11s. 9½d., New York currency, to Federal money.

```
   £56 11s.
     20          12|9,5
   ------          |-----
8)1131,7916         ,7916
  ---------
Ans. $141,4739+
```

Note.—1qr. is ,25d.
2qrs. are ,50d.
3qrs. are ,75d.
&c.

5. Reduce £35 11s. 6d., Connecticut currency, to Federal money. *Ans.* \$118 58c. 3m.+

6. Reduce £28 11s. 6d. Virginia currency, to Federal money. *Ans.* \$95,25.

7. Change £419 10s. 8½d., New York and Ohio currency, to Federal money. *Ans.* \$1048 83c. 8m.+

8. Reduce £721 9s. 11¼d., Massachusetts money, to Federal money. *Ans.* \$2404 98c. 9$\frac{5}{10}$m.+

9. Change £145, New York, &c., currency, to Federal money. *Ans.* \$362 50cts.

10. Change £134, New England currency, to dollars. *Ans.* \$446 66c. 6m.+

11. Reduce £14 6s. 4½d., New York, &c., currency, to Federal money. *Ans.* \$35 79c. 6$\frac{8}{10}$m.+

12. Reduce £9 11s. 4d., New England currency to Federal money. *Ans.* \$31 88c. 8$\frac{8}{10}$m.+

PROBLEM II.

To reduce New Jersey, Pennsylvania, Delaware and Maryland currencies to Federal money.

RULE.

Reduce the shillings, pence, &c., to the decimal of a pound; annex this decimal to the pounds; then multiply by 8 and divide the product by 3, the quotient will be dollars, &c.

EXAMPLES.

1. Reduce £41 19s., Pennsylvania currency, to Federal money.

```
       41,95
           8
    3)335,60
Ans. $111,86⅔
```

19s. reduced to the decimal of a pound, (by Inspection, see Rule, page 106,) are equal to £,95, &c.

A dollar, this currency, is 7s. 6d. =90d. and a pound =240d., and $\frac{90}{240}=\frac{3}{8}$; therefore a dollar is $\frac{3}{8}$ of a pound, and to divide by $\frac{3}{8}$, we × by 8 and ÷ by 3.

2. Reduce £14 6s. 8d., New Jersey, &c., currency, to Federal money. *Ans.* \$38 22c. 2m.+

3. Change £9 6s. 3d. to Federal money. *Ans.* \$24,833.

4. Reduce 19s. 11½d. to Federal money.

Ans. \$2 66c. 1m.+

5. Reduce £18 to Federal money. *Ans.* 48dols

6. Reduce 18 Pennsylvania shillings to Federal money.

Ans. 2dols. 40cts

7. Reduce £25 13s. 4½d. to Federal money.

Ans. 68dols. 45cts

PROBLEM III.

To reduce South Carolina and Georgia currency to Federal money.

RULE.

I. Reduce the shillings, pence, &c. to the decimal of a pound, as in problem 2. Then multiply by 30, and divide the product by 7; the quotient will be dollars, &c.

EXAMPLES.

1. Reduce £45 19s. Georgia currency, to Federal money.

$$\begin{array}{r} 45{,}95 \\ 30 \\ \hline 7)1378{,}50 \\ \hline \textit{Ans.}\ \$196{,}92\frac{6}{7} \end{array}$$

A dollar this currency is 4s. 8d.= 56d. and a pound=240d.; then $\frac{56}{240}=\frac{7}{30}$, therefore a dollar is $\frac{7}{30}$ of a pound and to divide by a fraction we must multiply by the denominator, and divide by the numerator.

2. Reduce £11 3s. 4d. to Federal money.

Ans. \$47 85cts. 6m. +

3. Reduce 18 Georgia shillings to dollars, &c.

Ans. \$3 857+

4. Change £9 18s. 6½. South Carolina currency, to Federal money. *Ans.* \$42 54 4+

5. How many dollars, &c. is there in 12s. 6d. South Carolina currency. *Ans.* \$2 67 8$\frac{4}{7}$.

6. Reduce £28 19s. to Federal money. *Ans.* \$124 07+

7. Reduce £94 14s. 8d. to Federal money.

Ans. \$405 99 8+

PROBLEM IV.

To reduce Canada and Nova Scotia Currency to Federal money.

RULE.

Reduce the shillings, &c. to the decimal of a pound as before; then multiply by 4, the quotient will be dollars, &c.

EXAMPLES.

1. Reduce £11 7s. Canada, &c. currency, to Federal money.

£11,35
4
———
Ans. $45 40

In this currency the dollar being 5s. a pound, is equal to 4 dollars, consequently if we multiply the pounds and decimals of a pound by 4, the product will be dollars and decimals of a dollar.

2. Reduce £28 9s. 6d. Canada currency, to dollars, &c. *Ans.* $113 90.

3. In £13 8s. Nova Scotia currency, how many dollars, &c. *Ans.* $53 60.

4. Reduce 16 Canada shillings to dollars, &c. *Ans.* $3 20.

5. Reduce £45 to dollars. *Ans.* $180.

6. Reduce £19 17s. 6¼d. Canada, &c. currency, to Federal money. *Ans.* $79 50,4.

PROMISCUOUS EXERCISES.

1. Reduce £5 6s. New England currency, to Federal money. *Ans.* 17 666+

2. Reduce £14 11 6d. New Jersey currency to dollars, &c. *Ans.* $38, 866+

3. Reduce £95 Pennsylvania currency, to dollars, &c. *Ans.* $253 33⅓.

4. Reduce 19 New York shillings to dollars, &c. *Ans.* $2 37½.

5. Reduce 18s. 6d. Georgia currency, to dollars, &c. *Ans.* $3 964+

6. Change £5 19s. 4½ North Carolina currency, to dollars, &c. *Ans.* $14 9218.

7. Change £14 10s. 6d. Massachusetts currency, to dollars, &c. *Ans.* $48 416+

8. In 135 shillings Connecticut currency, how many dollars, &c. *Ans.* $22 50

To Reduce Federal Money to the Currency of the several States.

I. To reduce Federal money to the currency of New England, Virginia, Kentucky and Tennessee, the dollar being $\frac{3}{10}$ = ,3 of a pound.

RULE.

Multiply the given sum by ,3, the product will be pounds and decimals of a pound.

EXAMPLES.

1. Reduce $125 41, to pounds, &c.

125 41	We multiply the dollars, &c. by ,3, and the product is pounds and decimals of a pound. We may then, if we choose, find the value of the decimal by inspection (see the rule, page 108) thus double 6 the first decimal figure is 12s. Then deduct 1 from the remaining figures, 23, leaves 22qrs., and 22qrs.= to 5d. 2qrs.
,3	
37,623	
20	
s. 12,460	
12	
d. 5,520	
4	
qrs. 2,080	*Ans.* £37 12s. 5d. 2qrs.

2. Reduce $14,565 to pounds, &c. *Ans.* £4 7s. 4d. 2qrs.+
3. Reduce $114,85 to pounds, &c. *Ans.* £34 9s. 1,2d.
4. Reduce $2,45 to shillings, &c. *Ans.* 14s. 8d. 1,6qr.

II. To Reduce Federal money to New York, North Carolina, and Ohio, currency, the dollar being $\frac{4}{10}$ of a £

RULE.

Multiply the given sum by ,4, the product will be pounds and decimals of a pound.

EXAMPLES.

1. Reduce $362 50 to pounds. *Ans.* £145.
2. Reduce $145 37½ to pounds, &c. *Ans.* £58 3s.
3. Reduce $18 75 to pounds, shillings, &c. *Ans.* £7. 10s.
4. Reduce $95 785 to pounds, &c. *Ans.* £38 6¼d.

III. To reduce Federal money to New Jersey, Pennsyl-

vania, Delaware and Maryland currency, the dollar being $\frac{3}{8}$ of a pound.

Multiply the given sum by 3, and divide the product by 8; the quotient will be pounds and decimals of a pound.

EXAMPLES.

1. Reduce \$18 75 to pounds, shillings, &c. *Ans.* £7 0s. 7,2d.
2. Reduce \$45 to pounds, &c. *Ans.* £16 17s. 6d.
3. Reduce \$95 62c. 8½m. to pounds, &c. *Ans.* £35 17s. 2¼d.

IV. To reduce Federal money to South Carolina and Georgia currency, the dollar being $\frac{7}{30}$ of a pound.

RULE.

Multiply the given sum by 7, and divide the product by 30; the quotient will be pounds and decimals of a pound.

EXAMPLES.

1. Reduce \$16 25 to pounds, &c. *Ans.* £3 15s. 9¾.
2. Reduce \$145 37½ to Georgia currency. *Ans.* £33 18s. 4¾.
3. Reduce \$19 75 to pounds, &c. *Ans.* £4 12s.
4. Reduce 75cts. to shillings, &c. *Ans.* 3s. 6d.

V. To reduce Federal money to Canada and Nova Scotia currency, the dollar being ¼ of a pound.

RULE.

Divide the given sum by 4; the quotient will be pounds and decimals of a pound.

EXAMPLES.

1. Reduce \$138½ to Canada currency. *Ans.* £34 12. 6d.
2. Reduce \$1184.50 to pounds. *Ans.* £296 2s. 6d.
3. Reduce \$75 32c. 9,5m. to pounds. *Ans.* £18 16s. 7¾d.
4. Reduce 90cts. to Canada currency. *Ans.* 4s. 6d.

Having shown how to reduce the old currencies of the several United States, and also those of Canada and Nova Scotia to Federal money, likewise to reduce Federal money to those currencies again, we now present the following Table, containing rules for changing each currency to the par of all the others. That is, for changing the currencies of the several States to their equal value in each of the other states. This is called domestic Exchange.

To use the following Table, first find the given currency at the top of the columns—then casting the eye down the column, until you come to the currency into which the given currency is to be changed and you will have the rule. Thus, if

be required to change New England currency to New York currency, we find
ew England currency at the top of the first or left hand column—then casting
e eye down that column until we come to New York currency, we have the
le, viz: add one third, and so for any other currency.

A TABLE CONTAINING RULES,

or reducing the Currencies of each of the United States, also Canada and Nova Scotia and Sterling Money, to the par of all the others.

o REDUCE *. England, Virginia, Kentucky, & Tennessee* Currency	TO REDUCE *New Jersey, Pennsylvania Delaware and Maryland*	TO REDUCE *New York, N. Carolina, and Ohio.*	TO REDUCE *S. Carolina and Georgia.*	TO REDUCE *Canada and Nova Scotia.*	TO REDUCE *Sterling.*
TO ennsylvan-a, N. Jersey, elaware, & aryland. dd $\frac{1}{4}$.	TO N. England, Virginia, Kentucky & Tennessee. Deduct $\frac{1}{5}$.	TO N. England, Virginia, Kentucky & Tennessee. Deduct $\frac{1}{4}$.	TO N. England, Virginia, Kentucky & Tennessee. × by 9 & ÷ by 7.	TO N. England, Virginia, Kentucky & Tennessee. Add $\frac{1}{5}$.	TO N. England, Virginia, Kentucky & Tennessee. Add $\frac{1}{3}$.
TO New York, . Carolina, nd Ohio. dd $\frac{1}{3}$.	TO New York, N. Carolina, and Ohio. Add $\frac{1}{15}$.	TO New Jersey, Pennsylvania, Delaware & Maryland. — $\frac{1}{16}$.	TO New Jersey, Pennsylvania, Delaware & Maryland. × by 45 & ÷ by 28.	TO New Jersey, Pennsylvania, Delaware & Maryland. Add $\frac{1}{2}$.	TO New Jersey, Pennsylvania, Delaware & Maryland. × by 5 & ÷ by 3.
TO S. Carolina Georgia. × by 7 & ÷ by 9.	TO S. Carolina & Georgia × by 28 & ÷ by 45.	TO S. Carolina and Georgia. × by 7 & ÷ by 12.	TO New York, N. Carolina, and Ohio. × by 12 & ÷ by 7.	TO New York, N. Carolina, and Ohio. × by 8 & ÷ by 5.	TO New York, N. Carolina, and Ohio. × by 16 & ÷ by 9.
TO anada and ova Scotia. × by 5 & ÷ by 6.	TO Canada and Nova Scotia. Deduct $\frac{1}{3}$.	TO Canada and Nova Scotia. × by 5 & ÷ by 8.	TO Canada and Nova Scotia. Add $\frac{1}{14}$.	TO S. Carolina and Georgia. — $\frac{1}{15}$.	TO S. Carolina and Georgia. Add $\frac{1}{27}$.
TO Sterling. educt $\frac{1}{4}$.	TO Sterling. × by 3 & ÷ by 5.	TO Sterling. × by 9 & ÷ by 16.	TO Sterling. — $\frac{1}{28}$.	TO Sterling. — $\frac{1}{10}$.	TO Canada and Nova Scotia. Add $\frac{1}{5}$.

Note. Some of the late writers on Arithmetic have thought proper to discon-
ue the Rules contained in the preceding Table for changing the old Currency
each of the United States to that of another; but as accounts were formerly
pt in these currencies, there may be some cases existing wherein it will be
cessary for men of business to practice these Rules, we therefore insert them,
gether with some examples

Application of the Rules contained in the preceding Table.

EXAMPLES.

1. Reduce £84 10s. 6d. New England currency into New York currency.

		£	s.	d.
See Rule by Table, viz: add $\frac{1}{3}$ to the given sum,		3)84	10	6
		28	3	6
	Ans.	112	14	0

2. Reduce £80 10s. 8d. New York currency, into New England currency. *Ans.* £60 8s.

3. Reduce £130 10s. 8d. New York currency, into Pennsylvania currency. *Ans.* £122 7s. 6d.

4. Reduce £428 10s. 6d. New Jersey currency, into New York currency. *Ans.* £457 1s. 10d.

5. Reduce £210 6s. 6d. New England currency, into Georgia currency. *Ans.* £163 11s. 8$\frac{1}{2}$d.+

6. Reduce £315 10s. 8d. Georgia currency, into New York currency. *Ans.* £540 18s. 3$\frac{3}{7}$d.

FOREIGN EXCHANGE.

The rates at which Foreign Coins are estimated in the United States, are exhibited in the following

TABLE.

Livre of France, - - - - - -	\$0,18$\frac{1}{2}$
Franc of France, - - - - - -	\$0,18$\frac{3}{4}$
Silver Rouble of Russia, - - - - -	\$0,75
Guilder or Florin of the United Netherlands, -	\$0,40
Mark Banco of Hamburgh, - - - -	\$0,33$\frac{1}{3}$
Real of Plate of Spain, - - - -	\$0,10
Real of Vellon of Spain, - - - -	\$0,05
Rix Dollar of Denmark, - - - -	\$1,00
Milrea of Portugal, - - - - - -	\$1,24
Tale of China, - - - - - -	\$1,48
Pagoda of India, - - - - -	\$1,84
Rupee of Bengal, - - - - -	\$0,50

EXAMPLES.

1. Reduce 5148 livres of France, to Federal money.

1 livre=18$\frac{1}{2}$cts.; then 5148×18$\frac{1}{2}$=\$952,38.

2. Reduce $952,38 to livres of France.

Thus, 95238cts.=190476 half-cts.; then 18½cts.=37 half-cents, and 190476÷37=5148 livres, *Ans.*

3. Reduce 987 Francs to Federal money.

Ans. $185,06¼.

4. Reduce 2540 Florins to Federal money. *Ans.* $1016.

5. Reduce $195,20 to Florins. *Ans.* 488.

6. Reduce 145 Tales of China to Federal money.

Ans. $214,60.

A Pound Sterling of Great Britain,	-	-		$4,44$\frac{4}{9}$.
A Pound Sterling of Ireland,	-	-	-	$4,10.
An English Guinea,	-	-	-	$4,66$\frac{2}{3}$.
An English Shilling	-	-	-	$0,22$\frac{2}{9}$.

RULE OF THREE DIRECT.

1. The Rule of Three Direct teaches from three given numbers to find a fourth, which shall have the same proportion to the third, as the second has to the first.

2. Of the given numbers, the first two are a supposition and the last (which may be known by the words, what cost? how much? how far? how many? &c.) is a demand.

3. In stating questions in the Rule of Three, two of the given numbers, that is, the first and third, must always be of the same name or kind; and the second or middle term must be of the same name or kind with the answer or term sought.

Sign. : : Four points set in the middle of four numbers, denote them to be proportional to one another by the Rule of Three, thus; 2 : 4 : : 8 : 16 is read, As 2 is to 4 so is 8 to 16.

RULE.

1. State the question so that the first and third terms will be of the same name or kind, and the second or middle term of the same name or kind with the answer.

2. Bring the first and third terms to the same denomina-

tion, and reduce the middle term to the lowest name mentioned in it.

3. Multiply the second and third terms together, and divide the product by the first term; the quotient will be the answer in the same denomination you left the middle term in; which may be brought into any other denomination required.

Proof.—The method of proof is by inverting the question.

EXAMPLES.

1. If 8 yards of cloth cost 12 dollars, what will 20 yards cost?

Operation.

yds.	dols.	yds.
8 :	12 : :	20
		12
	8)	240
	Ans.	$30

Here 20 yards, being a demand, must possess the third place; then 8 yards being the same kind, is the first term; and 12 dollars is the second term. Then the second and third terms multiplied together, and the product divided by the first term; the quotient is 30 dollars, which bears the same proportion to 20 yards, that 12 dollars does to 8 yards.

2. If $30 will buy 20 yards, how many yards will $12 buy?

dols.	yds.	dols.
30 :	20 : :	12
	12	
3,0)	24,0	
Ans.	8 yards.	

3. If 20 yards cost $30, what cost 8 yards?

yds.	dols.	yds.
20 :	30 : :	8
	8	
2,0)	24,0	
Ans.	12 dols.	

4. If 12 dollars will buy 8 yards of cloth, how many yards will 30 dollars buy?

dols.	yds.	dols.
12 :	8 : :	30
		8
	12)	240
	Ans.	20 yds.

It will be seen that the preceding sums are all similar.—The different results are made by changing the order of the terms.

5. If 3 cwt. of sugar cost $33 60, what will 12cwt. 2qrs. 14lbs. cost?

```
cwt.  $  cts.   cwt. qr. lbs.
 3 : 33  60 : : 12  2  14
112               4
---              --
336              50
                 28
                ---
                404
                101
               ----
               1414
               3360
              -----
              84840
             4242
            4242
           --------
    336)4751040(14140
         336
        ----
        1391
        1344
        ----
          470
          336
         ----
         1344
         1344
         ----
           00
```

State the question as before; then as the first and third terms must be of the same name, we reduce them both to lbs. Then multiply the second and third terms together, and divide the product by the first term, and the quotient or answer, is 141 dollars, 40 cents.

Note.—In multiplying and dividing dollars, cts. &c. observe the rules already taught for multiplying and dividing decimals, or Federal money.

2. When the first and third terms are Federal money, you may reduce them to the same denomination by annexing as many ciphers, (when necessary) as will make their decimal places equal. Then multiply and divide as in whole numbers, and the quotient or answer will be of the same name or denomination of the middle term.

6. If 5 dollars will buy 4 bushels of wheat, how many bushels can I buy for 118 dollars 75 cents.

```
cts.  bush.   cts.
500 : 4 : : 11875
              4 bush.
         ------
    500)47500(95Ans.
        4500
        ----
         2500
         2500
```

In this example, the first term being dollars, and the third term being dollars and cents, therefore, we reduce them both to cents, then multiply and divide as before.

7. If 3 pairs of stockings cost $1 75 what will 3 dozen pair cost? *Ans.* $21.

8. If 3 dozen pairs of stockings cost $21, what will 3 pairs cost? *Ans.* $1 75

9. If $12 75 buy 17 yards of cloth, how many yards can I buy for $2 25 at the same rate? *Ans.* 3 yards.

10. At 15cts. per lb. what will a firkin of butter come to, weight 54 lbs. *Ans.* $8 10.

11. If 6 horses eat 21 bushels of oats in one week, how many bushels will serve 20 horses the same time? *Ans.* 70 bushels.

12. Bought 6 chests of sugar, each weighing 8 cwt. 3 qrs., what do they come to at $8 50 per cwt.? *Ans.* $446 25.

13. If 12 months or one year's wages be $225 58, what will 5 months' wages be, at the same rate? *Ans.* $93 99 1$\frac{2}{3}$.

14. If a man lay out $211 25 in merchandise, and thereby gain $68 50, how much will he gain by laying out $25, at the same rate? *Ans.* $8 10 6$\frac{4}{10}$.

15. If I pay $71 50 for 11 barrels of flour, how many barrels can I have for $1014? *Ans.* 156 barrels.

16. What will 4 tierces of rice come to, each weighing 7cwt. 2qrs. 14lb., at $9,35 per cwt.? *Ans.* $285 17cts. 5m.

17. If a man's yearly income be $756, 65cts., what is that a day? *Ans.* $2, 07c. 3m.+

18. If 8 yards of cambric cost $5,48, what must be given for 25 pieces, each containing 27½ yards? *Ans.* $470, 93c. 7$\frac{5}{10}$m.

19. If 100 dollars in 1 year gain 6 dollars, what will 385 dollars gain in the same time? *Ans.* $23, 10c.

20. A owes B 1234 dollars, but not being able to pay the whole, B agrees that if he will pay him 57 cents on a dollar he will release him; how much must B receive? *Ans.* $703,38.

21. If 35 dollars will pay for 4cwt. of sugar, how much may be had for 215 dollars 19 cents at the same rate? *Ans.* 24cwt. 2qrs. 10lb. 6$\frac{3192}{3500}$oz.

22. If I give 1887 dollars for 148 acres of land, how many acres can I buy for $63,75 at the same rate? *Ans.* 5 acres.

23. When a tun of wine cost 140 dollars, what cost 3 quarts? *Ans.* 41cts. $6\frac{6}{10}$m.+

24. Bought a bale of cloth for 436 dollars 80 cents, at the rate of 7 dollars for 5 yards; how many yards did it contain? *Ans.* 312 yards.

25. A merchant bought 4 hogsheads of wine, containing 61, 63, $63\frac{1}{2}$ and 64 gallons, at 1 dollar 56 cents per gallon; what was the cost of the whole?

$61+63+63\frac{1}{2}+64=251\frac{1}{2}$gals. *Ans.* \$392,34.

26. If a man's income be 1296 dollars a year, and he spend 2 dollars $37\frac{1}{2}$ cents a day during the year, how much will he have saved at the year's end?

Ans. 429 dols. $12\frac{1}{2}$cts.

Sterling Money.

EXAMPLES.

27. If 3 yards of cloth cost 45s., how many yards may I have for £16 10s. at the same rate?

```
 s.  yds.   £.  s.
 45 : 3 : : 16 10
             20
            ---
            330
              3
         ------
      45)990(22yds.
         90
         --
          90
          90
          --
```

The first and third terms must be reduced to the same denomination, &c.

28. If 1 yard of cambric cost 12s., what cost 4 pieces, each measuring 20 yards? *Ans.* £48.

29. A grocer bought a hogshead of sugar, weighing 8 cwt. 3qrs., at 6d. per pound; what was the cost of it?

Ans. £24 10s.

30. How much sugar can I buy for £24 10s., at 6d. per pound? *Ans.* 8cwt. 3qrs.

31. If 58 yards of cloth cost £18 17s., what is that per Ell English? *Ans.* 8s. $1\frac{1}{2}$d.

32. A owes B £3475, but B compounds with him for 13s. 4d. on the pound; what must B receive for his debt?

Ans. £2316 13s. 4d.

33. A goldsmith sold a tankard for £10 12s. at 5s. 4d. per ounce; I demand the weight of it.

Ans. 3lb. 3oz. 15pwt

EXAMPLES FOR PRACTICE.

34. If a staff 5 feet long cast a shade on level ground 7 feet, what is the height of that steeple whose shade at the same time measures 171 feet? *Ans.* $122\frac{1}{7}$ft.

35. If $\frac{7}{8}$ of a ship cost $1960,25, what is the whole worth at that rate?

eighths. $ cts eighths

As 7 : 1960,25 : : 8 : *Ans.* $2240, 28cts. $5\frac{7}{10}$m.

36. If it take 22 bushels of wheat to make 4 barrels of flour, how many bushels will make 856 barrels?

Ans. 4708bush.

37. If 12 horses consume 30 bushels of oats in a week, how many bushels will serve 15 horses 3 weeks?

As 12 : 30 : : 15×3 : *Ans.* $112\frac{1}{2}$ bushels.

38. Bought of William Merchant 18cwt. 3qrs. of sugar at 8 dollars 12 cents per cwt., and gave him a note on James Paywell for £25 10s. (New England currency;) the rest I am to pay in cash; how much will make up the deficiency?

Ans. $67, 25cts.

39. Bought 50 pieces of kerseys, each 34 Ells Flemish, at 1 dollar $37\frac{1}{2}$ cents per Ell English; what did the whole come to? *Ans.* $1402,50.

40. A merchant bought 250 yards of cambric for $325\frac{1}{2}$ dollars; but finding it damaged, he is willing to loose 25 dollars by the sale of it: what must he demand per Ell English? *Ans.* $1 50c. $2\frac{1}{4}$m.

41. Sold a ship for $2540; I owned $\frac{2}{15}$ of her; what was my part of the money? *Ans.* $$338,66\frac{2}{3}$.

42. A merchant bought 18 pipes of wine, and is allowed 6 months credit, but for ready money he gets it 6 cents a gallon cheaper; how much does he save by paying ready money? *Ans.* $136,08.

43. Bought a hogshead of molasses for 13 dollars 75cts. and by accident 9 gallons leaked out; I then sold the remainder at 5 cents a pint; did I gain or loose, and how much? *Ans.* I gained $7,85.

44. Bought 240 yards of cloth at the rate of $7\frac{1}{2}$ dollars

for 5 yards, and sold it again at the rate of $11\frac{1}{2}$ dollars for 7 yards; did I gain or lose in the purchase; and how much?

Ans. I gained \$46, 28c. $5\frac{7}{10}$m.+

45. If a man have a salary of 327 English guineas (at 28s.) a year, how much may he spend one day with another, to lay up 625 dollars at the year's end?

Ans. \$2, 46c. $8\frac{4}{10}$m.

46. A merchant bought 14 pipes of wine at 14s. 6d., (New York currency,) per gallon; how many dollars will pay the purchase? *Ans.* \$3197,25.

47. A merchant bought 274 Ells Flemish of Holland, at 48 cents per Ell, and sold it again at 94 cents per Ell English; how much did he gain on the whole?

Ans. \$23, 1c. 6m.

48. If 50 gallons of water in one hour fall into a cistern which will hold 230 gallons, and by a pipe in the cistern 35 gallons run out in an hour; in what time will it be filled?

Ans. 15h. 20m.

49. A and B depart from the same place and travel the same road; but A goes 5 days before B at the rate of 20 miles a day; B follows at the rate of 25 miles a day; what distance must B travel to overtake A? *Ans.* 500 miles

---

RULE OF THREE INVERSE.

1. The Rule of Three Inverse teaches, from three given numbers to find a fourth, which shall have the same proportion to the second, as the first has to the third.

2. When more requires more, or less requires less, the question belongs to the Rule of Three Direct.

3. But if more requires less, or less requires more, the question belongs to the Rule of Three Inverse.

[More requiring more is when the third term is greater than the first, and requires the fourth term to be greater than the second; less requiring less is when the third term is less than the first, and requires the fourth term to be less than the second.—Also, more requiring less is when the third term is greater than the first, and requires the fourth term to be less than the second; and less requiring more is

when the third term is less than the first, and requires the fourth term to be greater than the second.]

Thus, if 3 men can perform a piece of work in 6 days, how long will it take 6 men to do the same? It is evident that if three men require 6 days, 6 men will do it in half the number of days, that is, 3 days Here more requires less, viz.: the more men the less time is required.

If 6 men require 3 days, how many days will 3 men require? Ans. It is evident that 3 men will require as much again time as 6 men would, viz.: 6 days. Here less requires more, that is, the less the number of men is, the more days are required.

RULE.

1. State the question, and reduce the terms the same as in the Rule of Three Direct.

2. Multiply the first and second terms together, and divide the product by the third, the quotient will be the answer in the same denomination that the middle term was reduced to.

EXAMPLES.

1. If 10 men can dig a ditch in 18 days, how many men will do the same in 6 days? *Ans.* 30 men.

2. If a man perform a journey in 10 days, when the days are 12 hours long, in how many days will he perform the same when the days are but 10 hours long? *Ans.* 12 days

3. If 20 bushels of grain, at 75 cents a bushel, will pay a debt, how many bushels, at 50 cents a bushel, will pay the same? *Ans.* 30 bushels.

4. What length of board that is 5⅓ inches wide will make a square foot? *Ans.* 27 inches.

5. If when wheat is $1,25 per bushel the penny-loaf will weigh 9oz., what ought it to weigh when wheat is $1,00 per bushel? *Ans.* 11oz. 5pwt.

6. If $100 in 12 months gain $6 interest, what principal will gain the same in 8 months? *Ans.* $150.

7. If 5 dollars will pay for the carriage of 3cwt. 150 miles, how far may 15cwt. be carried for the same money? *Ans* 30 miles.

8. How many men must be employed to finish in 8 days what 18 men would do in 24 days? *Ans.* 54 men.

9. There was a certain building erected in 8 months by 120 workmen, but the same being demolished, it is required to be rebuilt in 2 months; I demand how many men must be employed to do it? *Ans.* 480 men.

10. Suppose 850 soldiers are placed in a garrison, and their provisions calculated to last but 2 months; how many must leave the garrison, that the same provisions may last those who remain, 5 months? *Ans.* 510.

11. How many yards of cambric, $\frac{3}{4}$ yard wide, will line $5\frac{1}{2}$ yards of cloth which is $1\frac{1}{4}$ yards wide?

Ans. 9 yards 0qrs. $2\frac{2}{3}$ nails

12. A regiment of soldiers, consisting of 652 men, is to be clothed, each suit to contain $3\frac{1}{2}$ yards of cloth which is $1\frac{3}{4}$ yards wide, and lined with flannel $\frac{3}{4}$ yard wide; how many yards of flannel will line the whole?

Ans. 5324yds. 2qrs. $2\frac{2}{3}$ nails

Note.—The foregoing are the common methods of solving questions in the Rule of Three Direct and Inverse, which we consider most applicable to general practice.—There is, however, another method of solving questions in these Rules without making any distinction between Direct and Inverse proportion, and as some may have a preference for this method of operation, we therefore insert the following Rule, in which no distinction is made between Direct and Inverse proportion, together with some examples.

RULE.

Write that number for the 3d term which is of the same name or kind with the answer; then, if by the nature of the question the answer ought to be greater than the third term, write the greater of the two remaining numbers for the second term, and the less number for the first term. But if the answer ought to be less than the third term, place the least of the two remaining numbers for the second term, and the greater for the first term; then multiply the second and third terms together, and divide the product by the first term, and the quotient will be the answer in the same name of the third term

EXAMPLES.

1. If 6 yards cost $10, what will 12 yards cost?

Thus, as 6 yds. : 12 yds. : : 10 dols. :
Ans. 20 dollars.

2. If $20 will buy 12yds., how many yards will 10dols. buy?

20 dols. : 10 dols. : : 12 yds. :
Ans. 6 yards.

3. If 8 men can build a wall in 20 days, how many men will do the same in 5 days?

days. days. men.
As 5 : 20 : : 8 : *Ans.* 32 men.

4. If 32 men can build a wall in 5 days, how many men will do the same in 20 days?

days. days. men.
As 20 : 5 : : 32 : *Ans.* 8 men.

5. If 8 bushels of wheat cost 11 dollars, what will 25 bushels cost?

bush. bush. dols.
As 8 : 25 : : 11 :
11
8)275
Ans. $\$34\frac{3}{8}$.

6. If a man perform a journey in 11 days when the days are 15 hours long, in how many days will he perform the same when the days are but 10 hours long?

hours. hours. days.
As 10 : 15 : : 11 :
11
10)165
Answer $16\frac{1}{2}$ days.

7. How many yards of cloth which is $\frac{3}{4}$ yard wide will line 9 yards of broadcloth, which is $1\frac{1}{4}$ yards wide?

qrs. yds. yds.
As 3 : $1\frac{1}{4}$: : 9 : *Ans.* 15 yards

Note. When the first and second terms are of different denominations, they must both be reduced to the same denomination. And if the third term be a compound number, reduce it to the lowest name mentioned.

8. If 3cwt. 1qr. of sugar cost $21, 6cts., what cost 35 cwt.?

qrs. qrs. cts.
As 13 : 140 : : 2106 : *Ans.* $226,86.

9. If 45 bushels of grain, at 75 cents per bushel, will pay a debt, how many bushels, at 1 dollar per bushel, will pay the same? cts. cts. bush.

As 100 : 75 : : 45 : *Ans.* 33bu. 3pks.

Questions.

1. What does the Rule of Three Direct teach?
2. Of the three given numbers, what are the first two? What is the last?
3. In stating questions in the Rule of Three, which two numbers must always be of the same name or kind?
4. Of what kind must the middle term be?
5. What do four points set in the middle of four numbers denote?
6. What is the Rule for Direct proportion?
1. What does the Rule of Three Inverse teach?
2. When more requires more, or less requires less, to what Rule does the question belong?
3. If more requires less, or less requires more, to what Rule does the question belong?
4. What is the Rule for Inverse proportion?

PER CENTAGE.

The Latin term *per centum*, and its abbreviation *per cent*, signify by the hundred. Thus, 1 per cent of any number is $\frac{1}{100}$ of that number; 2 per cent is $\frac{2}{100}$; 3 per cent $\frac{3}{100}$; 4 per cent is $\frac{4}{100}$; 5 per cent is $\frac{5}{100}$; and so on.

1. What is 4 per cent of 1156 dollars?

$1156 4 per cent is $\frac{4}{100}$; therefore we multiply by 4 and divide by 100. To divide by 100, we need only cut off two figures from the right.

$1156	
4	
$46,24	

Ans. 46 dollars 24 cents.

2. What is 5 per cent or $\frac{5}{100}$, of $145 68 cents?

$145,68
5

Ans. 7'28,40=$7 28cts. 4 mills.

		$ cts. mls
3.	What is 12 per cent of $250?	*Ans.* 30.
4.	What is 7 per cent or $\frac{7}{100}$ of $500?	*Ans.* 35.
5.	Find 6 per cent of $96,48.	*Ans.* 5 78 8$\frac{8}{10}$
6.	Find 9 per cent of $216.	*Ans.* 19 44.
7.	What is 5$\frac{1}{2}$ per cent of $956, 14?	*Ans.* 52 58 7$\frac{7}{10}$
8.	What is 2$\frac{1}{4}$ per cent of $65,20?	*Ans.* 1 46 7.

INTEREST.

Interest is an allowance of so much per cent made by a debtor to a creditor, for the use of money.

In the New England States, the rate of interest is established by law at 6 per cent per annum; that is, \$6 for the use of \$100, or £6 for the use of £100, for one year; and in the same proportion for a greater or less sum, or for a longer or shorter time.

In New York, the lawful interest is 7 per cent, &c.

1. Principal is the sum due, for which interest is paid.
2. Rate is the sum per cent agreed on.
3. Amount is the principal and interest added together

Interest is of two kinds, *Simple* and *Compound*.

SIMPLE INTEREST.

Simple Interest is when interest is computed only on the original principal.

1. *To find the interest of any given sum for one year.*

RULE.

Multiply the principal by the rate per cent and divide the product by 100; the quotient will be the interest for one year.

Note. By this Rule are calculated Commission, Brokerage, Insurance, buying and selling Stock, &c.

EXAMPLES.

1. What is the interest of \$511 for one year, at 6 per cent?

Principal	\$511
Rate per cent	×6
Interest	30,66

We multiply by the rate per cent, and point off two figures for dividing by 100, and the answer is $30\frac{66}{100}$ dollars, or 30 dollars 66 cents.

2. What is the interest of \$1236, 38 cents for 1 year, at 7 per cent?

Principal	$1236,38
Rate per cent	×7
	86'54,66

After multiplying by the rate per cent, we place the separatrix as in Federal money. Then to divide by 100 we point off two figures, and the answer is 86 dollars $54\frac{66}{100}$ cents, or $86 54 cents $6\frac{6}{10}$m.

3. What is the interest of 341 dollars 37 cents for one year, at 5 per cent?

	$341,37
	5
	17'06,85

After multiplying by the rate per cent and placing the separatrix as in multiplication of Federal money, we need only call the figures on the left so many cents and those on the right decimals of a cent; thus 1706,85 cents = $17 6c. $8\frac{5}{10}$m.

3. Required the interest of $800 for one year, at $5\frac{1}{2}$ per cent. *Ans.* $44.

4. Required the interest of $956,25 for one year, at 7 per cent. *Ans.* $66 93c. $7\frac{5}{10}$m.

5. Required the interest of $1356 for one year, at 6 per cent. *Ans.* $81,36.

6. What is the interest of £500 for one year, at 5 per cent? *Ans.* £25.

7. Required the amount of $1528,25 for one year, at 4 per cent? *Ans.* $1589,38.

2. *To calculate interest for any number of years, months, &c*

GENERAL RULE.

1. Find the interest for one year as before.

2. Multiply the interest for one year by the given number of years.

3. For the months, take even parts of a year; thus, for 1 month take $\frac{1}{12}$ of a year, and for 2 months $\frac{2}{12}$ or $\frac{1}{6}$, &c.

4. For days, take even parts of a month, allowing it to contain 30 days; (or, if the sum be large and great accuracy required, work for the days by the Rule of Three Direct, by saying, as 365 days are to the interest for one year, so is the given number of days to the interest required.)

EXAMPLES.

1. What is the interest of $84,54 for 3 years 7 months and 10 days, at 6 per cent per annum?

		$84,54
		6
6 mo.=½ of 1 year.	½	507,24 interest for 1 year in cts. mills, &c.
		3
		1521,72 ditto for 3 years.
1 mo.=⅙ of 6 mo.	⅙	253,62 ditto for 6 months.
10 days=⅓ of 1 mo.	⅓	42,27 ditto for 1 month.
		14,09 ditto for 10 days.
Answer		1831,70 =1831cts. 7m. or $18, 31cts. 7m.

2. What is the interest of 210 dollars for 5 years, at 6 per cent per annum? *Ans.* $63.

3. Required the interest of $511,65 for 3 years and 4 months, at 5 per cent per annum. *Ans.* $85,27½.

4. Of $48,25 for two and a half years, at 6 per cent *Ans.* $7, 23c. 7$\frac{5}{10}$m.

5. Of 249 dollars for 3 years 9 months, at 6 per cent. *Ans.* $56, 2¼ cents.

6. Of $27,56 for 2 years, at 5 per cent. *Ans.* $2 75cts. 6m.

7. Of $951,17 for 4 years, at 5 per cent. *Ans.* $190 23cts. 4m.

8. What is the interest of $1 for 16 years 8 months, at 6 per cent per annum? *Ans.* $1.

9. What is the interest of $824,41 for 3 years 5 mos. and 10da., at 6 per cent per annum? *Ans.* $170,35cts. 9$\frac{2}{10}$m.+

10. What will £1825 amount to in 5 years, at 4 per cent per annum? *Ans.* £2190.

11. What is the interest of $145,96 for 3 years 9 mos., at 5¼ per cent per annum? *Ans.* $28, 73c. 5$\frac{8}{10}$m.

12. What will $15195 amount to in 5 years 4 months and 20 days, at 7 per cent? *Ans.* $20926, 89c. 1$\frac{6}{10}$m.+

13. What is the interest of $456 from April 15th, 1829, to March 5th, 1833, at 6 per cent? *Ans.* $106,40.

14. A note dated October 5th, 1831, for 256 dols. was paid,

principal and interest, March 20th, 1834: I demand the amount paid. Interest at 6 per cent. *Ans.* \$293,76

SHORT PRACTICAL RULES FOR CALCULATING INTEREST WHEN THE RATE IS 6 PER CENT.

1. *To find the interest for any number of months at 6 per cent.*

In casting the interest of any sum of money for 1 year, or 12 months, at 6 per cent, it is evident that the rate per cent. is just half the number of months in a year, or one-half per cent per month. Hence, for any term of time at 6 per cent, the rate being just $\frac{1}{2}$ per cent per month, it is evident that if we multiply the given sum by half the number of months, it will give the interest required.—Hence the following

RULE.

Multiply the principal by half the number of months, and divide the product by 100; the quotient will be the interest for the given time.

EXAMPLES.

1. What is the interest of \$148,21 for 1 year 4 months, or 16 months, at 6 per cent per annum?

Principal	\$148,21	After placing the separatrix in the product, we point off the 2 right hand figures (for dividing by 100.)
Half the months	8	

11′85,68 = \$11 85c. $6\frac{8}{10}$m.

2. What is the interest of \$225 for 3 years and 4 mos. at 6 per cent per annum? *Ans.* \$45,00.

3. What is the interest of \$95 34 for 9 months, at 6 per cent? *Ans.* \$4, 29+.

4. Required the interest of \$211,25 for 7 months, at 6 per cent. *Ans.* \$7, 39c. $3\frac{7}{10}$m.+

5. Required the interest of \$137 for 2 months, at 6 per cent? *Ans.* \$1, 37c.

6. What is the interest of £125 for 8 months, at 6 per cent? *Ans.* £5.

2. *To calculate interest for months and days, at 6 per cent.*

RULE.

Divide the principal by 2, placing the separatrix as in division of Federal money; the quotient will be the interest for 1 month, in cents, &c.

Multiply the interest for one month by the given number of months and decimal parts of a month, or for the days take even parts of a month.

EXAMPLES.

1. What is the interest of \$135,28 for 7 months and 15 days, or 7½ months?

	2)135,28	or thus	2)135,28
Interest for 1 mo.	67,64		67,64
	.7½		7, 5
do. for 7 mos.	473,48		33820
do. for ½ mo.	33,82		47348
Ans.	507,30c.		507,300c. =\$5
=5dols. 7c. 3 mills.			[07c. 3m.

2. What is the interest of \$133,24 for 2 years 7 months and 10 days, or 31⅓ months, at 6 per cent per annum?
Ans. \$20 87c. $4\frac{2}{10}$m.

3. Required the interest of \$428 for 20 days, or ⅔ of a month, at 6 per cent. *Ans.* \$1 42c. 6m.+

4. What is the interest of \$256,18 for 1 year 5 months and 15 days, or 17½ months, at 6 per cent?
Ans. \$22 41 $5\frac{7}{10}$m.

Note. As there are many short methods for casting interest at 6 per cent, it may sometimes be convenient, when the rate is any other than 6 per cent, to cast the interest first at 6 per cent, and add thereto, or subtract, such part as the rate is more or less than 6 per cent. Hence, to calculate interest at 7 per cent, we have the following concise and practical

RULE FOR THE STATE OF NEW YORK.

Add to the interest at 6 per cent, ⅙ part of itself, the sum will be the interest at 7 per cent.

EXAMPLES.

1. What is the interest of \$148 for 1 year 5 months, or 17 months, at 7 per cent?

Principal \$148
Half the months $= \times 8\frac{1}{2}$
11,84
74
Int. at 6 per cent $\frac{1}{6}$ | 12,58
2,09,6
Int. at 7 per cent. 14,67,6 + = 14 dols. 67c. 6m.+

2. What is the interest of \$125,40 for 1 year and 8 months, at 7 per cent?

125,40
,10 $= \frac{1}{2}$ the mos.
add $\frac{1}{6}$)12,5400 int. at 6 pr.ct.
2,09
Ans. \$14,63

3. Required the interest of \$148 for 3 years and 4mos. at 7 per cent per annum.

Princ. \$148
20 $\frac{1}{2}$ the months.
add $\frac{1}{6}$|29,60 int. at 6 per ct.
4,93,3
Ans. \$34,53,3 = \$34,53c, 3+

4. Required the interest of \$95,44 for 10 days, or $\frac{1}{3}$ of a month, at 7 per cent.

\$ cts.
2)95,44
$\frac{1}{3}$)47,72 int. for 1 mo. [at 6 per cent.
add $\frac{1}{6}$|15,90 do. for $\frac{1}{3}$ mo.
2,65
18,55 = 18c. $5\frac{5}{10}$m.

COMMISSION, BROKERAGE, INSURANCE, &c.

1. Commission is an allowance of so much per cent to a correspondent or factor abroad, for buying and selling goods for his employer.

2. Brokerage is an allowance of so much per cent to persons assisting merchants and traders, in buying or selling goods.

3. Insurance is a premium at so much per cent, allowed to persons and offices for making good the loss of vessels, houses, merchandize, &c., which may happen from storms, fire, &c. The instrument which binds the parties is called a Policy.

EXAMPLES.

1. What must I demand for selling goods to the amount of 548 dollars, at $2\frac{1}{4}$ per cent commission? *Ans.* \$12,33

2. Required the commission on 875 dollars 56 cents, at 3½ per cent? *Ans.* $30 64c. 4$\frac{6}{10}$m.

3. What must my correspondent demand, who has sold goods to the amount of 3500 dollars 24 cents, at 3⅞ per cent commission? *Ans.* $135 63c. 4$\frac{3}{10}$m.

4. What is the brokerage upon 511 dollars 50 cents, at 33⅓ per cent? *Ans.* $170,50

5. What may a broker demand who sells goods to the amount of $4500 at 12½ per cent? *Ans.* $562,50.

6. What is the insurance of $350, at 2½ per cent? *Ans.* $8,75.

7. A man's house, estimated at 2450 dollars, is insured against fire for 3⅓ per cent per year; what insurance does he pay annually? *Ans.* $81 66c. 6m.+

8. What is the insurance of a ship and cargo, valued at $69450, at 15½ per cent? *Ans.* $10764¾.

STOCK is the general name given to all monies invested in trading companies, or of any corporation or fund of Government.

When $100 of Stock sells for $100, it is said to be at par? when it sells for more, it is said to be above par; and when it sells for less, it is said to be below par.

9. What is the value of $5421 of stock, at 1 12½ per cent; that is, when $1 of stock sells for $1,12½, which is 12½ per cent above par, or 12½ per cent advance? *Ans.* $6098 62½.

10. What is the value of $950 bank stock at 95 per cent, that is 5 per cent below par? *Ans.* $902 50.

11. What is the value of $1230 insurance stock, at 118¾ per cent, that is 18¾ per cent above par? *Ans.* $1460 62½.

SIMPLE INTEREST BY DECIMALS.

This is by many considered the best and most concise way of computing Interest. In this method of casting interest, (both simple and compound) the rate per cent is expressed in a decimal fraction, [illegible] when so expressed, is called the ratio.

TABLE OF RATIOS.

Rate per cent.	Ratio.	Rate per cent.	Ratio.	Rate per cent.	Ratio.
$\frac{1}{4}$	,0025	2	,02	5	,05
$\frac{1}{2}$	,005	3	,03	$5\frac{1}{2}$	,055
$\frac{3}{4}$	,0075	4	,04	6	,06
1	,01	$4\frac{1}{2}$	,045	7	,07

Thus the interest of 1 dollar for one year, at 1 per cent, is 1 cent, or the $\frac{1}{100}$ part of 1 dollar; which expressed in a decimal fraction, is the ,01 of a dollar, or the ratio of one per cent; and $\frac{1}{4}$ of ,01 is ,0025or the ratio of $\frac{1}{4}$ per cent. The $\frac{1}{2}$ of ,01, is 005, or the ratio of $\frac{1}{2}$ per cent; and $\frac{3}{4}$ of ,01, is ,0075, or the ratio of $\frac{3}{4}$ per cent. So, likewise, the interest of $1 for one year, at 6 per cent, is 6 cents, or $\frac{6}{100}$ of one dollar, which written in a decimal fraction, is ,06, or the ratio of 6 per cent.; and so any other per cent expressed in a decimal fraction, is the ratio of that rate per cent as appears by the foregoing Table; from which to find the interest of any sum of money for any given rate and time, we have the following

RULE.

Multiply the principal, ratio, and time, continually together, (observing to place the decimal point the same as in multiplication of decimals) and the last product will be the interest required.

EXAMPLES.

1. Required the interest of $315 36 for 4 years, at 6 per cent per annum.

```
 $315 36 principal.
      ,06 ratio.
 ----------
 18,9216 Interest for 1 year
     ×4 by time.
 ----------
 75,6864 Ans. $75 68cts. 6 4/10m.
```

2. What is the interest of $537 48 for $3\frac{1}{2}$ years, at $5\frac{1}{2}$ per cent per annum. *Ans.* $103 46 $4\frac{9}{10}$.

3. What is the amount of $736 58 for $8\frac{3}{4}$ years, at 6 per cent per annum. *Ans.* $386 70 $4\frac{5}{10}$.

CASE II.

The amount, time, and ratio, given to find the principal.

RULE.

Multiply the ratio and time together, and add 1 to the product for a divisor, by which divide the amount, and the quotient will be the principal required.

EXAMPLES.

1. What principal will amount to $1138,2150 in 5 years, at 6 per cent per annum?

$,06\times5+1=1,30)1138,2150(\$875,55$ *Ans.*

2. What principal will amount to $2844,82650 in $7\frac{1}{2}$ years, at 7 per cent per annum. *Ans.* $1865,46.

3. What principal, at $6\frac{1}{2}$ per cent per annum, will amount to $2216,10, in 12 years. *Ans.* $1245.

CASE III.

The amount, principal, and time, given, to find the ratio.

RULE.

Subtract the principal from the amount, and divide the remainder by the product of the time and principal, and the quotient will be the ratio required.

EXAMPLES.

1. At what rate per cent will $850,22 amount to $1156,2992 in 6 years? *Ans.* 6 per cent.

From the amount $1156,2992
Subtract the principal 850,22

$\$850,22\times6=5101,32)306,0792(,06=6$ per cent. *Ans.*
306,0792

0

2. At what rate per cent will $325,00 amount to $390,00 in 4 years? *Ans.* ,05, or 5 per cent.

3. At what rate per cent will $450, amount to $698,625 in $8\frac{1}{2}$ years? *Ans.* ,065, or $6\frac{1}{2}$ per cent.

4. At what rate per cent will $381 gain $240,03 interest in 9 years.

Note. In this question it will be seen that the principal, interest, and time, are given, to find the rate per cent. Therefore, if we divide the interest by the product of the principal and time, the quotient will be the rate per cent.

Thus, 381 × 9 = 3429(240,03(07 = 7 per cent. *Ans.*

5. At what rate per cent will $500 gain $127,50 interest in 8½ years? *Ans.* 3 per cent.

CASE IV.

The principal, rate per cent, and amount given, to find the time.

RULE.

Subtract the principal from the amount, and the remainder will be the interest.

2. Divide the interest by the product of the ratio and principal, and the quotient will be the time.

EXAMPLES.

1. In what time will $325 amount to $390, at 5 per cent per annum?

$$\begin{array}{r} \$390,00 \\ 325,00 \\ \hline 325 \times 05 = 16,25)65,00(4 \text{ years, } Ans. \\ 65,00 \end{array}$$

2. In what time will $450 amount to $698,625, at 6½ per cent, or ,065? *Ans.* 8 ,5 or 8½ years.

3. In what time will $381 amount to $621,03, at 7 per cent? *Ans.* 9 years.

4. In what time will $500 gain $127,50 interest at 3 per cent? 500 × ,03 = 15,00)127,50(8,5 = 8½ years.

5. In what time will $856,17 gain $222,6042 at 6 per cent per annum? *Ans.* 4⅓ years.

CASE V.

To calculate interest for days.

RULE.

Multiply the principal by the ratio and that product by the given number of days, and divide the last product by 365, (the number of days in a year,) and you will have the interest required.

EXAMPLES.

1. What is the interest of $341 for 250 days, at 6 per cent?

```
        $341
         ,06
       20,46
         250
      102300
      4092
365)5115,00(14,013+=$14 1c. 3m.+
```

2. What is the interest of $125 for 135 days, at 5½ per cent per annum? *Ans.* $2 54c. $2\frac{8}{10}$m.+

3. What is the interest of $525 for 73 days, at 7 per cent? *Ans.* $7,35

CASE VI.

To compute interest on notes or obligations, when there are payments in part, or endorsements.

RULE.

1. Find the amount of the whole principal for the whole time.

2. Find the amount of each payment from the time it was paid to the time of settlement; and lastly deduct the amount of the several payments from the amount of the whole principal.

EXAMPLES.

1. For value received, I promise to pay William Merchant, or order, five hundred dollars, with interest. July 1st, 1828. JAMES PAYWELL.

Endorsements.

Received, February 1, 1829, $158.
" April 11, 1830, $225.
" June 6, 1831, $ 50.

How much remained due, April 16th, 1832, interest at 6 per cent?

Principal on interest from July 1st, 1828,	$500 00
Interest to April 16th, 1832, 3yr. 9m. 15d. (45½m.)	113 75
Whole amount of principal,	$613 75

First payment Feb. 1, 1829,	$158,000
Interest to April 16th, 1832, (38½m.)	30,415
Second payment, April 11, 1830,	225,000
Interest to April 16th, 1832, (24⅙m.)	27,187
Third payment, June 6, 1831,	50,000
Interest to April 16th, 1832, (10⅓m.)	2,583
Amount of the payments deducted,	$493,185

Ans.—Remains due, April 16th, 1832, $120,565

2. On demand, for value received, I promise to pay Peter Trusty, or order, six hundred and fifty dollars with interest.

May 11th, 1829. TIMOTHY CAREFUL.

$650 *Endorsements.*

June 1, 1830, received	$241 25
April 16, 1831,	$125 50
September 6, 1832	$208 00

How much remained due on said note, January 16th, 1834 interest at 6 per cent? *Ans.* $167 62c. 6m

$1000. *New London, September* 20, 1830.

For value received, I promise to pay James Judson, or order, one thousand dollars, on demand, with interest.

JAMES RICHARDSON.

Endorsements.

October 15, 1831, received,	$500.
January 20, 1832,	$225.
April 10, 1833,	$158.
March 5, 1834,	$177,50.

How much remains due on said note, February 25th, 1835, allowing interest at 6 per cent? *Ans.* $34 55c. 9m.

The foregoing rule for computing interest on notes and obligations where endorsements have been made, may be considered incorrect, from the fact that it does not proceed on the ground of the annual payment of interest. But it may be considered correct, where notes or obligations are given on interest, payable at a future period, without any conditions of paying the interest annually. Thus, if A give B his note on interest, for $100, payable at some future period, without any condition of paying the interest yearly, B cannot demand any part of the note, or interest, until the time arrives when the whole becomes due; and in this case, if A advances to B any part of said note or interest, by way of endorsement, before said note becomes due, he is entitled to interest on said endorsement, from the time it was paid, up to the time when the note becomes due; because in this case, B will have the use of A's money before it becomes due, and ought, therefore, to pay interest on the same.

But when notes are given payable at a future period with interest annually, this rule will not be correct. For if A gives B his note for $100, for a term of years on interest, at 6 per cent. to be paid annually at the end of each year, and A accordingly pays the yearly interest of $6, and has it endorsed thereon, (which according to the tenor of A's note is then justly due to B) it is evident that A would not be entitled to any interest on these endorsements of the yearly interest, at the time of final settlement. For A is bound to pay this yearly interest by the tenor of his note. It is therefore evident that in this case, the foregoing rule will not give a correct result.

CONNECTICUT RULE.

Established by the Supreme Court of the State of Conn.

"Compute the interest to the time of the first payment, if that be one year or more from the time the interest commenced, add it to the principal, and deduct the payment from the sum total; if there be after payments made, compute the interest on the balance due to the next payment, and then deduct the payment as above; and in like manner, from one payment to another, till all the payments are absorbed; provided, the time between one payment and another be one year, or more. But if any payment be made before one year's interest has accrued, then compute the interest on the principal sum due on the obligation, for one year; add it to the principal, and compute the interest on the sum paid, from the time it was paid, up to the end of the year; add it to the sum paid, and deduct that sum from the principal and interest added as above.*

"If any payments be made of a less sum than the interest, arisen at the time of such payment, no interest is to be computed, but only on the principal sum for any period."

EXAMPLES.

1. A note, dated January 4th, 1829, was given for $1000, on interest, at 6 per cent, and there were payments endorsed on it as follows, viz:

1st	payment,	February 19th, 1830,	$200.
2d	"	June 29th, 1831,	$500.
3d	"	November 14th, 1831,	$360.

* If a year does not extend beyond the time of final settlement: but if it does, then find the amount of the principal remaining unpaid, up to the time of settlement: likewise the amount of the sum paid, from the time it was paid, up to the time of final settlement, and deduct this amount from the amount of the principal. But if there be several payments made within said time, find the amount of the several payments, from the time they were paid, to the time of settlement, and deduct the sum of the several amounts, from the amount of the principal.

I demand how much remains due on said note, the 24th of December, 1832.

$1000,00	principal of the note.
67,50	interest to February 19th, 1830, (13½mo.)
1067,50	amount.
200,00	1st payment deducted.
867,50	due February 19th, 1830.
70,845	interest to June 29, 1831, (16⅓mo.)
938,345	amount.
500,000	second payment deducted.
438,345	balance due, June 29th, 1831.
26,30	interest for one year.
464,645	amount for one year.
373,50	amount of 3d payment to June 29, 1832, (the end of the year) being 7½ months.
91,145	due June 29th, 1832.
2,657	interest to Dec. 24th, 1832, (5m. 25d.)
93,802	balance due on said note, Dec. 24th, 1832.

MASSACHUSETTS RULE.

"Compute the interest on the principal sum to the first time when a payment was made, which, either alone or together with the preceding payment, (if any,) exceeds the interest then due; add that interest to the principal, and from the sum subtract the payment, or the sum of the payments made at that time, and the remainder will be a new principal, with which proceed as with the first principal; and so on to the time of settlement.

$800. *March* 1, 1822.

1. For value received, I promise to pay William Stanby, or order, eight hundred dollars, with interest.

JAMES PAYWELL

Endorsements.

May 1, 1823,	received	$300
June 16, 1824,	. . . .	$ 90
September 17, 1825,	. . . . .	$ 12
December 19, 1825,	. . . . . .	$ 16
March 1, 1826,	. . . .	$108
October 16, 1827,	. . . . . .	$350

What was there due, August 31, 1828, interest at 6 per cent? *Ans.* $115,77+

Operation.

The principal,		$800,00
Interest from March 1, 1822, to May 1, 1823, (14 mo.)		56,00
Amount,		$856,00
Payment May 1, 1823, a sum greater than the interest,		300,00
Due May 1, 1823, forming a new principal,		$556,00
Int. of $556 from May 1, 1823, to June 16, 1824, (13½mo.)		37,53
Amount,		$593,53
Payment June 16, 1824, a sum greater than the interest due,		90,00
Due June 16, 1824, forming a new principal,		$503,53
Int. on $503,53 from June 16, 1824, to March 1, 1826, (20½mo.)		51,61
		$555,14
Pay't Sept. 17, 1825, less than the interest then due	$ 12	
Pay't Dec. 19, 1825, " " "	16	
Pay't March 1, 1826, a sum greater than the interest,	108	
		$136,00
Due March 1, 1826, forming a new principal,		$419,14
Int. from March 1, 1826, to Oct 16, 1827, (19½mo.)		40,86
		$460,00
Pay't Oct. 16, 1827, a sum greater than the interest then due,		350,00
Due Oct. 16, 1827, forming a new principal,		$110,00
Interest from Oct. 16, 1827, to August 31, 1828, being the time of settlement, (10½ months.)		5,77
Balance due, August 31, 1828.		$115,77+

$500. *January* 1, 1830.

2. For value received, I promise to pay Peter Trusty, or order, five hundred dollars, with interest.

William Payson.

On this note were the following *Endorsements.*

April 1, 1830,	received	$40
October 14, 1830, ...		$ 8
March 1, 1831, ...		$12
April 1, 1831, ...		$30

How much remains due August 16, 1831, interest at 6 per cent? *Ans.* $455,57.

COMPOUND INTEREST

Is that which arises both from the *principal* and *interest*; that is, when the interest is added to the principal at the end of the year, and on that amount the interest cast for another year and added as before; and so on for any number of years.

RULE.

Find the interest for 1 year, and add it to the principal; call this the amount for the first year. Find the interest of this amount and add it thereto for the amount of the second and so on for any number of years.

Subtract the original principal from the last amount, and the remainder will be the Compound Interest.

EXAMPLES.

1. What is the amount, and what the compound interest, of $150 for 3 years, at 6 per cent per annum?

Operation.

```
 $150=1st principal.
    6
 ----
 9,00=interest.
150    =1st principal added.
------
159,00=amount for the 1st year, or 2d principal.
     6
------
 9,5400=interest.
159,    =2d principal added.
--------
168,5400=amount for the 2d year, or 3d principal.
       6
---------
10,112400=interest.
```

(Carried over.)

[Operation brought up.]

10,112400=interest.
168,54

\$178,652400=amount for 3 years=\$178 65cts. 2,4m.
150, =first principal subtracted.

\$ 28,652400=compound int. for 3yrs.=\$28 65c. 2,4m.

2. What will \$125 amount to in 4 years, at 6 per cent per annum, compound interest? *Ans.* \$157,80c. 9m+

3. What is the compound interest of \$750 for 5 years, at 5 per cent? *Ans.* \$207,21+

4. What will be the amount of \$410,50 for 3 years, at 6 per cent? *Ans.* \$488,912+

5. What is the compound interest of \$500 for 3 years, at 6 per cent? *Ans.* \$95,50c. 8m.

6. What is the amount of \$1000 for 4 years, at 6 per cent per annum, compound interest?

Ans. \$1262, 47c. $6\frac{96}{100}$m.+

Note. When there are months, or months and days, in the question, first find the amount for the years, and on that amount calculate the interest for the months, or months and days; this interest added to the amount for the years will give the amount required.

7. What will \$148,25 amount to in 3 years and 6 months, at 6 per cent? *Ans.* \$181,865+

8. What is the compound interest of \$500 for 4 years 2 months and 15 days, at 5 per cent? *Ans.* \$114,08+

BY DECIMALS.

RULE.

Multiply the principal continually by the amount of \$1 or £1 for one year, at the given rate per cent, until the number of multiplications is equal to the given number of years. Thus, at 5 per cent, the amount for 1 year is 1,05; at 6 per cent, it is 1,06, &c.

1. What will be the amount of \$500 for 3 years, at 6 per cent per annum?

Thus, 500×1,06×1,06×1,06=\$595,508, *Ans.*

Or, find the amount of \$1, for 3 years, at 6 per cent per annum.

1,06 amount for 1 year.
1,06
636
106
1,1236 amount for 2 yrs.
1,06
67416
11236
1,191016 amount for 3 yrs.

Note. Since the amount of 2 dols. will be just twice as much as 1 dollar, and 3 dollars 3 times as much as 1 dollar, and so on, it is evident that if we multiply the amount of \$1 by any given number of dollars, we have the amount for that number of dollars. Thus, the amount of \$1 for 3 years, at 6 per cent, is 1,191016 ; this, multiplied by \$500, gives the amount \$595,508, the same as in example 1st.

Hence we may have a Table showing the amount of £1 or \$1, for any number of years, by which we may easily find the amount of any sum for the same time, by multiplying the amount taken from the Table by the given sum.

A TABLE,

Showing the amount of \$1, *or* £1, *for any number of years, not exceeding* 20, *at* 5 *and* 6 *per cent, Compound Interest.*

YEARS.	5 PER CENT.	6 PER CENT.	YEARS.	5 PER CENT.	6 PER CENT.
1	1,05	1,06	11	1,71034	1,89829
2	1,1025	1,1236	12	1,79585	2,01219
3	1,15762	1,19101	13	1,88565	2,13292
4	1,21550	1,26247	14	1,97993	2,26090
5	1,27628	1,33822	15	2,07893	2,39655
6	1,34009	1,41851	16	2,18287	2,54727
7	1,40710	1,50363	17	2,29201	2,69277
8	1,47745	1,59384	18	2,40661	2,85433
9	1,55132	1,68947	19	2,52695	3,02559
10	1,62889	1,79084	20	2,65329	3,20713

3. What is the amouut, and what the Compound Interest, of \$350 for 5 years, at 6 per cent ?

Am't of \$1 for 5yrs. at 6 per cent, by the Table 1,33822
Multiply by the principal, - - - - - ×350
6691100
401466
Answer. Amount for 5 years, - - =468,37700
Subtract the principal, - - - - - - 350
Leaves the Compound Interest, - - \$118,377

4. What is the amount, and what the Compound Interest of $311,25 for 6 years, at 5 per cent?

Ans. { $417,103 Amount.
{ $105,85,3 Compound Interest.

[The learner may work all the preceding examples in Compound Interest by the foregoing Table of Amounts, if necessary for further practice.]

Note. Any sum at 6 per cent, Simple Interest will double itself in 16 years 8 months; and at Compound Interest, in 11 years 8 months and 22 days.

Questions.

What does the term per centum, or per cent, signify?

What is Interest?

What is the Rate of interest, established by law in the New England States?

What is the lawful interest in New York State?

What is the Principal?—Rate?—Amount?

What is Simple Interest?

How do you find the interest of any sum for 1 year?

What is the general Rule for calculating interest for any number of years, months, &c.?

How do you find the interest of any sum for any number of months at 6 per cent?

How do you calculate interest for months and days, at 6 per cent?

What concise Rule have we for calculating interest for New York State?

What is Commission?—Brokerage?—Insurance?—Stock?

When is Stock said to be at par? When above par? When below par?

When the rate per cent is expressed in a Decimal Fraction, what is it called?

Havinng the amount time and rate per cent given, how do you find the principal?

Having the amount, principal and time given, how do you find the rate per cent?

Having the principal, rate per cent and amount given, how do you find the time?

How do you compute the interest on Notes, or obligations where endorsements have been made?

What is Compound Interest?

What is the Rule for Compound Interest?

DISCOUNT

Is an allowance made for the payment of any sum of money before it becomes due. After the discount is deducted, the remainder is the present worth; or, such a sum as, if put at interest, at the given rate and time, would amount to the given sum.

RULE.

1. As the amount of $100, or £100, at the given rate and time, is to the interest of 100, at the same rate and time, so is the given sum to the discount.

Subtract the discount from the given sum, and the remainder is the present worth.—Or by

2. Divide the given sum by the amount of $1, or £1, at the given rate and time, the quotient will be the present worth. Subtract the present worth from the given sum, and the remainder is the discount.*

EXAMPLES.

1. What is the present worth of $132,50, due 1 year hence, at 6 per cent discount?

[By Rule 1.]

```
     $     $     $  cts.
As 106 : 6 : : 132,50
                    6
                 ------
                   $ cts.
  106,00)795,00(7,50 dis.
         74200
        -------
  132,50  53000
Dis. 7,50  53000
  ------  -----
Ans. 125,00 present worth.
```

[By Rule 2.]

The amount of $1 for 1 year, at 6 per cent, is $1,06.

```
Therefore 1,06)132,50(125 Ans.
               106
               ---
                265
                212
                ---
                 530
                 530
                 ---
```

2. What sum, in ready money, will discharge a debt of £540, due 3 years 4 months hence, at 6 per cent?

Am't of £1 for 3 years 4 months, at 6 per cent, } £ 1,20)£ 540,00(£ 450 Ans.

3. What is the present worth of 500 dollars due 2 years 6 months hence, at 5 per cent?

$ 1,125)$ 500,000($ 444,44,4+ Ans. (cts. m.)

4. What is the present worth of $1080, due 5 years 10 months hence, at 6 per cent? *Ans.* $800.

5. What is the discount of $460, due 2 years 6 months hence, at 6 per cent? *Ans.* $60.

6. Bought goods to the amount of $1260,50 on eight months credit: how much ready money must I pay to discharge the same, discounting at 6 per cent?

Ans. $1212, 1c. 9m.+

Note. When sundry sums are payable at different times, find the present worth of each payment separately, and then add them together.

---

* This is precisely the same as Case II, Simple Interest by Decimals, since the present worth of any sum is such a principal as, if put at interest for the given rate and time, would amount to said sum, it follows that we have the amount, time and rate per cent given, to find the principal, or present worth.

7. What is the present worth of $1000, one-half payable in 8 months, and the other half payable in 8 months after that, at 6 per cent discount? *Ans.* $943, 73c. 2m.+

8. What is the discount of $1384, of which $500 are payable in 1 year 4 months, and the remainder at the end of 4 years, at 6 per cent? *Ans.* $208, 13c. 3m.+

EQUATION OF PAYMENTS.

Equation of Payments is the method of finding the mean time to pay at once several debts, due at different times.

RULE.

Multiply each payment by the time at which it is due, then divide the sum of the products by the sum of the payments, and the quotient will be the answer.

EXAMPLES.

1. A owes B $380, to be paid $100 in 6 months, $120 in 7 months, and $160 in 10 months; what is the equated time for the payment of the whole debt

$$\begin{array}{r} 100\times 6=600 \\ 120\times 7=840 \\ 160\times 10=1600 \\ \hline \end{array}$$

$380)3040(8 months. *Ans.*

2. A merchant owes $500, payable as follows; $150 in 4 months; $200 in 6 months, and the rest at 8 months, and he is to make one payment of the whole; I demand the equated time. *Ans.* 6 months.

3. A owes B $150, to be paid in 6 months; $180 to be paid in 8 months; $200 to be paid in 10 months, and $250 to be paid in 12 months; what is the equated time for the payment of the whole? *Ans.* $9\frac{16}{39}$ months.

4. What is the equated time for the payment of $1200 of which $500 are payable in 10 months, $400 in 20 months, and the rest in $2\frac{1}{2}$ years? *Ans.* $18\frac{1}{3}$ months.

5. A merchant owes $1800, to be paid $\frac{1}{3}$ in 5 months, $\frac{1}{6}$ in 10 months, $\frac{1}{6}$ in 18 months, and the rest in 20 months,

andwishes to pay the whole at once; I demand the equated time *Ans.* $13\frac{1}{3}$ months.

Questions.

1. What is Discount?
2. What is the present worth of any sum?
3. (Rules.) How do you find the Discount of any sum?
4. How do you find the present worth?
5. When several sums are payable at different times, how do you find the present worth?

1. What is Equation of Payments?
2. What is the rule for Equation of Payments?

ANNUITIES AT SIMPLE INTEREST.

An annuity is a sum of money payable every year, or for a certain number of years, or forever.

When the annuity is not paid at the time it becomes due, it is said to be in arrears.

The sum of all the annuities, together with the interest due on each for the time they have been forborne, is called the amount.

1. *To find the amount of an Annuity.*

RULE.

1. Cast the interest on the given annuity for one year, then for two years, three years, four years, &c., up to the given time, less 1. Then multiply the annuity by the given number of years, and add the product to the whole interest, and the sum will be the amount required.

EXAMPLES.

1. If an annuity of 200 dollars per annum remain unpaid, (that is, in arrears,) 6 years, what is the amount due, reckoning interest at 6 per cent.?

The interest of \$200 for 1 year is . .	\$12,00
" " " 2 years is . .	24,00
" " " 3 years is . .	36,00
" " " 4 years is . .	48,00
" " " 5 years is . .	60,00
6 years annuity at \$200 per year is . .	1200,00
	Ans. \$1380,00

2. If a salary of $325 per year remain unpaid, or in arrears, for the term of 5 years, what amount is then due, interest at 6 per cent? *Ans.* $1820.

3. If a man, having an annual pension of $75 a year, receive no part of it until the end of 7 years, what is the amount due, interest at 5 per cent? *Ans.* $603,75.

2. *When an annuity is bought off, or paid all at once, at the beginning of the first year, the sum that is paid for it is called the present worth.*

To find the present worth of an annuity at Simple Interest.

RULE.

Find the present worth of each year by itself, discounting from the time it falls due, and the sum of all the present worths will be the present worth required.

EXAMPLES.

1. What is the present worth of an annuity of $500, to continue 4 years, at 6 per cent per annum?

As 106 : 100 ::	$500 :	$471,6981+	pres. worth	1st year.
112 : 100 ::	500 :	446,4285+	"	2d year.
118 : 100 ::	500 :	423,7288+	"	3d year.
124 : 100 ::	500 :	403,2258+	"	4th year.

Ans. $1745,0812=$1745, 8cts. $1\frac{2}{10}$m.

2. What sum of ready money is equal to an annuity of 100 dollars to continue 3 years, discounting at 6 per cent? *Ans.* $268,371.

3. What is 600 dollars yearly rent, to continue 4 years, worth in ready money at 6 per cent? *Ans.* $2094, 09½c.

4. What is the present worth of an annual salary of 800 dollars, to continue 5 years, at 6 per cent? *Ans.* $3407 51,2c.

Questions.

1. What is an annuity?
2. When is an annuity said to be in arrears?
3. What is the sum of all the annuities, together with their interest, called?
4. How do you find the amount of an annuity?
5. What is the present worth of an annuity?
6. How do you find the present worth of an annuity?

TO FIND THE NET WEIGHT OF ANY GOODS OR MERCHANDIZE, WHERE TARE IS ALLOWED.

This is a part of the rule formerly called Tare and Tret, the remainder of which is now become obsolete in the United States, there being no such allowances as Tret, Cloff, &c. now made use of. We shall here notice,

1. Gross weight, which is the whole weight of any kind of goods, including the box, cask, bag, &c. that contains them.

2. Tare, which is an allowance made for the box, cask, bag, &c., which contains them, and is either at so much per box, bag, &c., or at so much per cwt., or at so much on the whole gross weight.

3. Net, which is what remains after the Tare is taken out.

CASE I.

If the question be an Invoice, add all the gross weights into one sum, and all the tares into another sum; then subtract the whole tare from the whole gross, the remainder will be the net.

EXAMPLES.

1. What is the net weight of 4 hogsheads of sugar marked with the gross weights and tares, as follows?

	cwt.	qrs.	lbs.		lbs.
No. 1, —	9	2	14	Tare	105
No. 2, —	8	3	20	"	87
No. 3, —	9	1	17	"	95
No. 4. —	8	2	13	"	84
Whole gross,	36	2	8		371lb.tot. tare.
Whole tare 371lb.=	3	1	7		
Ans.	33	1	1 net.		

2. What is the net weight of 5 tierces of rice, No. and weight as follows?

	cwt.	qrs.	lbs.		lbs.
No. 1, —	4	3	15	Tare	41
No. 2, —	4	1	12	"	50
No. 3, —	3	3	21	"	48
No. 4, —	4	2	22	"	47
No. 5, —	4	1	25	"	45

Ans. Net 20cwt. 1qr. 4lbs

CASE II.

When the tare is at so much for each box, cask, bag, &c. that contains the goods, multiply the tare of one by the number of boxes, bales, &c., and subtract the product from the gross, the remainder will be the net.

EXAMPLES.

1. In 5 hhds. of sugar, each weighing 9cwt. 3qrs. 17lbs. gross, tare 78lbs. per hogshead, how much net?

	cwt.	qrs.	lbs.	
	9	3	17	gross of 1 hogshead.
			5	
	49	2	1	gross weight of 5 hogsheads.
$78 \times 5 =$	3	1	26	whole tare of 5 hogsheads.
Ans.	46	0	3	net.

2. What is the net weight of 12 tubs of butter, each weighing 1cwt. 2qrs. 11lbs., tare per tub 17 pounds?

Ans. 17cwt. 1qr. 12lbs.

Note. It has now become a general custom and law, to call 100lbs. of almost every article of merchandize a hundred weight, instead of 112lbs. as formerly practiced, and in the following examples this custom is adopted.

3. In 285 barrels of raisins, each weighing 1cwt. 3qrs. 15lbs., tare per barrel 16lbs., how many pounds net?

Ans. 49590lbs.

4. In 17 bags of coffee, each weighing 97 pounds 5oz., tare per bag 3 pounds 7oz., how much net?

Ans. 15cwt. 3qrs. 20lbs. 14oz.

5. What is the net weight, and value, of 8 hogsheads of tobacco, each 9cwt. 2qrs. 21lbs. gross, tare per hogshead 96 pounds, net at 8$\frac{3}{4}$ cents per pound?

Ans. Net 7000 pounds. Value \$612,50.

6. Sold 15 hogsheads of sugar, each weighing 8cwt. 3qrs. 17lbs., tare per hogshead 45lbs.; required the net weight, and its value at 7$\frac{3}{4}$ cents per pound.

Ans. Net 12705. Value \$984,63$\frac{3}{4}$.

CASE III.

When the tare is at so much per cwt., divide the given quantity by the aliquot parts of a hundred weight for the

tare, which subtract from the gross the remainder will be the net.

EXAMPLES.

Note. The examples in this Case are predicated on the modern custom of counting 100lbs. to the cwt.

1. In 10 casks of rice, each 4cwt. 2qrs. 15lbs., tare per cwt. 10lbs., what is the net?

	cwt.	qrs.	lbs
	4	2	15
			10
10 pounds$=\frac{1}{10}$)	46	2	00
	—4	2	15
Ans.	41	3	10 Net.

2. What is the net weight of 9 barrels of potash, each weighing gross 2cwt. 3qrs. 18lbs., tare $12\frac{1}{2}$lbs. per cwt.?
Ans. 23cwt. 0qr. 7lb. 6oz.

3. Bought 43cwt. 2qrs. 20lbs. of sugar, tare per cwt. 10 pounds; required the net weight, and its value at 8dols. 33cts. per cwt.
Ans. { Net 39cwt. 1qr. 8lbs.
Value \$327, 61c. $8\frac{9}{10}$m.

4. What is the net weight, and value, of 24 barrels of figs, each 1cwt. 3qrs. 22lbs., tare per cwt. $12\frac{1}{2}$lbs., at 7 cents per pound?
Ans. { Net 41cwt. 1qr. 12lbs.
Value \$289, 59cts.

5. In 83cwt. 3qrs. 15lbs. gross, tare per cwt. 15lbs., what is the net weight? *Ans.* 71cwt. 1qr. $6\frac{1}{2}$lbs.

Questions.

Are there any such allowances now made as "Tret and Cloff"?
1. What is Gross weight?
2. What is Tare?
3. What is Net?

I. If the question be an Invoice, how do you find the net weight?
II. When the tare is so much per box, cask, bag, &c., how do you find the net weight?
III. When the tare is at so much per cwt., how do you find the net weight?

BARTER

Is the exchanging of one commodity for another; and directs merchants and traders how to make an exchange without loss to either party.

RULE.

Find the value of the article whose quantity is given; then find what quantity of the other article, at the proposed rate can be bought for the same money.

EXAMPLES.

1 What quantity of sugar, at 11cts. per pound, must be given for 11lbs. of indigo, at \$2,50 per pound?

First see what 11lbs. of indigo will come to at \$2,50 per pound.—Thus, \$2,50×11=\$27,50. Then \$27,50cts. ÷11cts.=250lbs. *Answer.*

2. How much tea, at \$,38 per lb., must be given for 24bush. of wheat, at \$1,25 per bushel? *Ans.* 78$\frac{18}{19}$lbs.

3. How much salt, at \$1,50 per bushel, must be given for 15bush. of oats, at \$,37½ per bushel? *Ans.* 3bu. 3pks.

4. A man sold 356lbs. of pork, at 8 cents per pound, for which he received 10 dollars in money, and the rest in corn at \$,75 per bushel; how much corn did he receive? *Ans.* 24$\frac{16}{25}$ bushels.

[Find what 356lbs. of pork will come to, then deduct the \$10 from that sum; then find how many bushels may be had for the remainder.]

5. How much rice, at \$4,68 per cwt., must be given for 3½ hundred weight of raisins, at 8cts. per pound? *Ans.* 6cwt. 2 quarters 22 $\frac{58}{117}$ pounds.

6. A and B bartered, A had 250lbs. of sugar, at 11cts. per pound, for which B gave him 11lbs. of indigo; what was the indigo rated at per pound? *Ans.* \$2,50.

7. A delivered B 3 hogsheads of brandy at \$1,08 per gallon, for 135 yards of cloth; what was the cloth per yard? *Ans.* \$1, 51cts. 2 mills.

8. A farmer delivered to a merchant 5 cords of wood, at \$6,50 per cord; in pay he received 16 gallons of molasses at \$,37½ per gallon, 2 barrels of flour at \$6,25 per barrel, 2yds. of broadcloth at \$4,37½ per yd., and the rest in money; how much money did he receive? *Ans.* \$5¼.

The ready money and barter price of one article, and the ready money price of the other given, to find its barter price.

9. A has linen cloth worth \$,42 per yard, ready money, but in bartering he will have \$,50 per yard; B has broad-

cloth worth \$4,50 per yard, ready money; at what price ought B to rate his broadcloth per yard to be in proportion to A's bartering price?

cts. cts. cts. cts.
As 42 : 50 : : 450 : $535\frac{5}{7}$=\$5, $35\frac{5}{7}$cts. Therefore, B must rate his broadcloth at \$5,$35\frac{5}{7}$ per yard, to be in proportion to A's bartering price.

10. A has 240 bushels of rye, at \$0,90 per bushel, which he barters with B at \$0,95 per bushel, for wheat worth \$1,12 per bushel, ready money; what ought B to rate his wheat at per bushel to be in proportion to A's bartering price, and how many bushels of wheat must A receive for his 240 bushels of rye?

Ans. { Bartering price of B's wheat \$1,$18\frac{2}{9}$.
A must receive $192\frac{6}{7}$ bushels.

LOSS AND GAIN

Is a rule by which merchants and traders discover their profit or loss in trading. It also instructs them how to rise or fall in the price of their goods, so as to gain or lose so much per cent, &c.

EXAMPLES.

1. Bought 35 yards of broadcloth for \$131, 25cts., and sold it again at \$4, 25cts. per yard; what did I gain on the whole? *Ans.* \$17,50.

2. Bought a hogshead of sugar, containing $8\frac{3}{4}$cwt. for \$67,60, and sold it out at $10\frac{1}{2}$cts. per pound; what did I gain on the whole? *Ans.* \$35,30.

3. Bought 125 yards of cambric for \$62, 50cts., and sold it again at \$0, 45cts. per yard; how much did I gain or lose on the whole? and how much on each yard?

Ans. { Lost on the whole \$6,25,
On each yard \$0,05.

4. Bought a chest of tea, weighing 75lbs., at $31\frac{1}{2}$cts. per lb. and sold it again at 42cts. per pound; did I gain or lose, and how much? *Ans.* Gained \$7,$87\frac{1}{2}$.

5. Bought a hogshead of rum containing 120 gallons, at 56cts. per gallon: paid for duties \$6,00, and for carting \$2,00, and by accident 10 gallons leaked out; at what rate

must I sell the remainder per gallon, to gain $25 on the whole? *Ans.* $0, 91c. $0\frac{2}{10}$m.+

2. *To know how a commodity must be sold, to gain or lose so much per cent.*

RULE.

Multiply the given sum by the rate per cent, written as a decimal fraction, and to the product add the given sum for the gain per cent, but subtract for the loss per cent.

EXAMPLES.

1. If I buy wine at $1,50 per gallon, how must I sell it per gallon, to gain 25 per cent?

$$\begin{array}{r} \$1,50 \\ ,25 \\ \hline 750 \\ 300 \\ \hline ,3750 \\ +\ 1,50 \\ \hline \end{array}$$

Answer $1,8750=$1,87½.

2. Bought broadcloth at $3,75 per yard, but being damaged I am willing to sell it so as to lose 12 per cent; what must I sell it at per yard? *Ans.* $3,30.

3. Bought tea at 54 cents per pound; how must I sell it per pound so as to gain 12½ per cent? *Ans.* $0,60,7½m.

4. Bought a hogshead of sugar weighing 8½cwt. at $7 50cts. per cwt.; how much must I sell the whole for, to gain 30 per cent? *Ans.* $82, 87c. 5m.

5. If 350bbls. of flour cost $2275, what must it be sold per barrel to gain 15 per cent? *Ans.* $7,47½.

6. Bought 120 gallons of wine at $1,08 per gallon, but by accident 10 gallons leaked out; at what rate must I sell the remainder per gallon, to gain upon the whole prime cost at the rate of 9 per cent? *Ans.* $1, 28c. $4\frac{2}{10}$m.+

3. *The prices at which goods are bought and sold being given, to find the rate per cent of gain or loss.*

RULE.

First see what the gain or loss is by subtraction; then as the price it cost is to the gain or loss, so is $100 to the gain or loss per cent

Or, make the gain or loss the numerator, and the prime cost the denominator of a common fraction, which, reduced to a decimal, will give the gain or loss per cent.

EXAMPLES.

1. If I buy cloth at \$0,80 per yard, and sell it again at \$1,00 per yard, what do I gain per cent?

Sold for \$1,00
Cost ,80

Gain per yard, ,20

cts.		cts.		\$
As 80	:	20	: :	100

100

80)2000(25 per cent. *Ans.*
160
400
400

Or thus:
Gain per yard $\frac{20}{80}$ reduced
Prime cost
to a Decimal Fraction, by annexing ciphers to the numerator and dividing by the denominator, (see Reduction of decimals,) gives ,25, or 25 per cent, the same as before.

2. A merchant bought goods to the amount of \$845,36, and sold the same for \$951,03; what did he gain per cent on the purchase price?

\$ cts.		\$ cts.		\$ cts.		\$ cts.	
As 845,36	:	105,67	: :	100,00	:	12,50	or $12\frac{1}{2}$ per cent. *Ans.*

Sold for \$951,03
Prime cost 845,36
Whole gain \$105,67

Or thus, $\frac{10567}{84536}=,125=12\frac{1}{2}$ per ct.

3. If I buy a hundred weight of sugar for \$8,00, and sell it again at $9\frac{1}{4}$ cents per pound; what do I gain per cent? *Ans.* $29\frac{1}{2}$ per cent.

4. A grocer bought a hogshead of rum for \$84, and by accident so many gallons leaked out that he sold the remainder for \$67,20; what did he lose per cent?

Ans. ,20, or 20 per cent.

5. If I buy cloth at \$2,00 per yard, and sell it again for \$2,50 per yard, what should I gain by laying out 100 dollars? *Ans.* \$25.

6. A merchant bought indigo at \$1,20 per pound, and sold it again for \$100,80 per hundred weight; what was lost per cent.? *Ans.* 25 per cent

7. At 25 cents profit on a dollar, how much per cent.? *Ans.* 25 per cent. At ,30? At ,12½? At ,15? At ,18¾? At ,6¼? At ,0575, or ,05¾? At ,215, or, ,21½? &c.

Note. When goods are bought or sold on credit, we must find, by discount, the present worth of their price in order to calculate the true gain or loss.

8. Bought goods to the amount of \$442, on 8 months credit, and sold the same again for \$478,125, ready money; what did I gain per cent, discounting on the purchase price at 6 per cent?

Thus, the amount of 1 dollar at 6 per cent, for 8 mos. = 1,04) 4 4 2, 0 0 (\$ 4 2 5. Therefore the present worth of the purchase price is 425 dollars.

Sold for	\$478,125
Prime cost	425,000
Whole gain	53,125

Then, $\frac{\text{gain } 53,125}{\text{Prime cost } 425,000}$ reduced to a decimal fraction = ,125 = 12½ per cent.

Or, As \$425 : \$53,125 : : \$100 : \$12,50, or 12½ per cent.

9. A merchant bought broadcloth at \$4 per yard, ready money, and sold the same again at \$4,251 on 18 months credit; did he gain or lose, and what per cent, allowing 6 per cent discount on the selling price?

Ans. He lost ,025 = 2½ per cent.

4. *The price sold for, and gain or loss per cent given, to find the cost of any article.*

RULE.

As \$100, with the gain per cent added, or loss per cent subtracted, is to the price, so is 100 to the prime cost.

EXAMPLES.

1. Sold a hogshead of molasses at \$0,30 per gallon, and thereby gained 25 per cent; what did the hogshead cost me?

```
  $    cts.   $
As 125 : 30 : : 100
             100
     ----------
     125)3000(24c. cost
         250   [per gallon.
         ---
          500
          500
          ---
```

then, 1hhd. = 63 gals.

```
  ,24
  ---
  252
 126
 ---
```

Ans. \$15,12 cost of [the hogshead.

2. Sold goods for $951,03, by which I gained 12½, or $12,50, per cent; what did the goods cost me?

Ans. $845,36.

3. A merchant sold indigo at $100,80 per cwt., and thereby lost 25 per cent; what did it cost him per lb.?

As 75 : 100,80 : : 100 : $134,40 cost per cwt.

Then $134,40÷112=$1,20 *Ans.*

4. Sold 350bbls. of flour for $2616,25 by which I gained at the rate of 15 per cent; what did it cost per barrel?

Ans. $6. 50 cents.

Questions.

What is Barter?
What is the Rule?
What is Loss and Gain?
What does it instruct merchants and traders to do?
II. How do you find how a commodity must be sold so as to gain or lose so much per cent?
III. Having the prices at which goods are bought and sold, how do you find the rate per cent, of gain or loss?
IV. Having the price sold for, and gain or loss per cent given, how do you find the cost of any article?

FELLOWSHIP

Is a rule by which persons trading in partnership adjust their accounts so that each may have his share of the gain, or sustain his share of the loss, in proportion to his share of the stock.—Also, by this rule a bankrupt's estate may be divided among his creditors.

SINGLE FELLOWSHIP

Is when the several shares of stock are continued in trade an equal term of time.

RULE.

As the whole stock, is to the whole gain or loss, so is each man's share of the stock, to his share of the gain or loss.

EXAMPLES.

1. Three men, A, B and C, traded in company; A put in $200, B $400, and C $600; they gained $348: what was each man's share of the gain?

A's stock $200		\$ \$ \$ \$	
B's stock 400		1200 : 348 : : 200 : 58	A s gain
C's stock 600		1200 : 348 : : 400 : 116	B's gain
		1200 : 348 : : 600 : 174	C's gain
Whole stock 1200			

In this example, the whole stock being $1200, A's stock $200=$\frac{1}{6}$ of the whole, thus $\frac{200}{1200}=\frac{1}{6}$; B's stock $400=$\frac{1}{3}$ of the whole, and C's stock $600=$\frac{1}{2}$ of the whole; then $\frac{1}{6}$ of $348=$58, A's part: $\frac{1}{3}$ of $348=$116, B's part, and $\frac{1}{2}$ of $348=$174, C's part, as before.

Or, we may first find the gain on $1

Thus, \$1200 : \$348 : : \$1 : \$0,29; then A's stock 200×,29 =$58, A's gain; B's stock $400×,29=$116, B's gain; and C's, stock $600×,29=$174, C's gain.

Proof. It is evident that if the several shares of gain or loss, added together, be equal to the whole gain or loss the work is right. Thus, $58+$116+$174=$348, the whole gain.

2. Three merchants, E, F and G, gained by trading in company $300; E's stock was $300, F's $600, and G's $900; what was the gain on $1, and how much was each man's share of the gain? *Ans.* The gain on $1 is $$\frac{1}{6}$; then, $\frac{1}{6}$ of $300=$50, E's part; $\frac{1}{6}$ of $600=$100, F's part, and $\frac{1}{6}$ of $900=$150, G's part of the gain.

3. A and B trade in company; A puts in $1250, and B $850. They find they have gained $700; what is the gain on $1, and what is each one's share of the gain?

Ans. $$\frac{1}{3}$. and A's $416,666+ B's $283,333+

4. Four men, A B C and D, shipped 1150 barrels of flour; A put on board 150 barrels, B 200 barrels, C 500 barrels, and D 300 barrels; in a storm the seamen threw overboard 325 barrels; what number of barrels did each lose? *Ans.* A lost $42\frac{18}{46}$, B $56\frac{24}{46}$, C $141\frac{14}{46}$, D $84\frac{36}{46}$.

5. Two men, C and D, join their stock and trade in company; C put in $650 and D $550. They gained $300 what is each man's share of the gain?

Ans. C's $162,50, D's $137,50.

6. A bankrupt is indebted to A $375, to B $125,50, to C $110, to D $695,50, and to E $294, and his whole es-

tate amounts to only $1200, which he gives up to his creditors; what must each receive in proportion to his claim?

Ans. A $281,25, B $94,12½, C $82,50, D $521,62½, and E $220,50.

7. Divide $1850 among 5 persons so that their shares shall be to each other as 1, 2, 3, 4, 5.

Ans. $123,33⅓, $246,66⅔, $370, $493,33⅓, $616,66⅔.

8. Two partners, C and D, trade with a capital of $3500; C put in $1950 and D the rest. By misfortune they lose $700; what was D's stock, and how much is each one's share of the loss?

Ans. D's stock $1550, and C's loss $390, D's $310.

9. Two merchants, E and F, traded in company and gained $875. E put in $1150, F put in so much that his share of the gain was $300; what was F's stock, and what was E's share of the gain?

Ans. F's stock $600. E's gain $575.

10. Two men, B and C, traded in company with a joint capital of $1350. They gained $450, of which B took $300, and C the remainder; what was each one's stock?

Ans. B's stock $900. C's $450.

11. C, D and E drew a prize of $2500, of which C is to have 5 shares, D 8 shares and E 7 shares; what is each man's part? *Ans.* C $625, D $1000, E $875.

COMPOUND FELLOWSHIP.

When the several stocks, &c. of the partners are continued in trade an unequal term of time, it is called Compound, or Double Fellowship.

RULE.

Multiply each man's stock, or share, by the time it was continued in trade.

Then, as the sum of the products, is to the whole gain or loss, so is each man's product, to his share of the gain or loss.

EXAMPLES.

1. Two men hired a pasture for $24; A put in 6 cows for 4 months, and B 12 cows for 3 months; what ought each to pay?

Six cows for 4 months are the same as 24 cows for one month, for $6\times4=24$; and 12 cows for 3 months are the same as 36 cows for 1 month, for $12\times3=36$. Hence the question is the same as if A had put in 24 cows for 1 month, and B 36 cows for 1 month.

$$\left.\begin{array}{lr} & 6\times4=24 \\ & 12\times3=36 \\ & \text{---} \\ \text{Sum of the products} & 60 \end{array}\right\} \begin{array}{l} \quad\ \$ \qquad\qquad\quad \$ \\ 60 : 24 :: 24 : \ \ 9,60 \text{ A's.} \\ 60 : 24 :: 36 : 14,40 \text{ B's.} \end{array}$$

2. Three merchants enter into a partnership; B puts in $300 for 4 months, C $150 for 5 months, and D $200 for 8 months. They gain $400; what is each man's share?

Ans. { B's. C's. D's.

3. Three persons hire a pasture in company for which they pay $96. C put in 100 sheep for 2 months, D put in 300 sheep for 5 months, and E 450 sheep for 4 months, what must each pay?

Ans. { C. D. E.

4. Two men, B and C, gained by trading $250. B's stock was $350 for 8 months, C's $640 for 5 months; required each man's share of the gain?

Ans. { B's share C's share.

5. Three persons received $665 interest. B's principal was $4000 for 12 months, C's $3000 for 15 months, and D's $5000 for 8 months; what is each man's share?

Ans. B's $240. C's $225. D's $200.

6. B and C enter into partnership for 16 months. B at first put in $500 and at the end of 12 months he took out $300; C at first put in $300, and at the end of eight months he put in $500 more. They gained $525; required each man's particular share of the gain?

Ans. { B's $$228,84\frac{8}{13}$. C's $$296,15\frac{5}{13}$

7. A commenced trade on the first of January with $850 and on the first of March he took in B with $560, and on he first of May he took in C with $650; at the end of the

year they find they have gained \$950; what must each man share of the gain?

$$Ans. \begin{cases} A\ \$461,42\frac{18}{21}. \\ B\ \$253,33\frac{7}{21}. \\ C\ \$235,23\frac{17}{21}. \end{cases}$$

Questions.

What is Fellowship? What is Single Fellowship? What is the Rule for Single Fellowship? What is Compound, or Double Fellowship? What is the Rule for Double Fellowship?

VULGAR FRACTIONS.

For the principal definitions of Fractions, the scholar will refer to those already given, Page 86.

I. Any number that will divide two or more numbers without a remainder, is called a common measure, or divisor; and the greatest number that will do this is called the greatest common measure, or divisor. Thus, 4 is a common divisor of 8, 16 and 24, because it will divide each without a remainder; but the greatest common divisor of these numbers is 8.

II. The common multiple of two or more numbers is that number which can be divided by each of those numbers without a remainder, and the least number that can be so divided is called the least common multiple. Thus, 24 is a common multiple of 3, 4 and 6; but their least common multiple is 12.

III. When two or more fractions have the same denominator, it is called their common denominator.

PROBLEM I.

To find the greatest common divisor of two numbers

RULE.

Divide the greater number by the less; and this divisor by the remainder, and so on, always dividing the last divisor by the last remainder till nothing remain; and the last divisor will be the common divisor.

EXAMPLES.

1. What is the greatest common measure of 91 and 117, or in other words, what is the greatest number that will divide 91 and 117 separately without a remainder?

Operation.

91)117,(1
    91
    26)91(3
        78
        13)26(2
            26

We divide the greatest number by the least, and the remainder is 26; therefore, 91 is not a factor of 117; then 26 in 91 will go 3 times, and 13 remains. Hence 26 is not a factor of 91; then 13 will go in 26, 2 times, and nothing remains. Hence, 13 is the greatest common divisor of 91 and 117, or the greatest number that 91 and 117 can be divided by, without a remainder.

2. What is the greatest common measure of 48 and 56? *Ans.* 8.

3. What is the greatest common divisor of 132 and 144? *Ans.* 12.

4. What is the greatest common divisor of 148 and 236? *Ans.* 4.

5. What is the greatest common divisor of 1224 and 1080? *Ans.* 72.

Note. If the greatest common divisor of more than 2 numbers is required, find the common divisor of two of them first; then of that common divisor, and one of the other numbers, and so on.

6. What is the greatest common divisor of 144, 192, and 456?

Thus, 144)192(1
        144
        48)144(3
            144

48 = greatest common divisor of 144 and 192. Then

48)456(9
    432
    24)48(2
        48

Ans. 24.

This Rule is also useful in finding a common divisor to divide the terms of a fraction by, in order to reduce them to their lowest terms.

7. Find the greatest common divisor of the terms of the fraction $\frac{108}{132}$, and by it reduce the fraction to its lowest terms. *Ans.* $\frac{9}{11}$.

8. Reduce $\frac{95}{152}$ to its lowest terms. *Ans.* $\frac{5}{8}$.

9. Reduce $\frac{96}{544}$ to its lowest terms. *Ans.* $\frac{3}{17}$.
10. Reduce $\frac{2568}{5136}$ to its lowest terms. *Ans.* $\frac{1}{2}$.

PROBLEM II.

To find the least common multiple of two or more numbers.

RULE.

1. Divide by any number that will divide two or more of the given numbers without a remainder, and set the quotients, together with the undivided numbers, in a line beneath. Divide this line as before, and so on until no two of the numbers can be divided by the same divisor; the continued product of all the divisors and numbers in the last line, will be the least common multiple required.

EXAMPLES.

1. What is the least common multiple of 2, 4, 3 and 6?

Operation.

$$\begin{array}{r|cccc} 2) & 2 & 4 & 3 & 6 \\ \hline 3) & 1 & 2 & 3 & 3 \\ \hline & 1 & 2 & 1 & 1 \end{array}$$

Ans. $2\times3\times2=12$

In dividing the numbers 2, 4, 3 and 6 by 2, we find that 2 is not contained in 3 even; therefore we bring the 3 down with the quotients. We then divide again by 3, and bring down the 2, which 3 will not divide, and the divisors and quotients are factors, which multiplied together produce a common multiple; and since these factors are expressed in the least numbers, it is evident that it is their least common multiple.

2. What is the least common multiple of 6 and 8? *Ans.* 24

3. What is the least common multiple of 6 and 18? *Ans.* 18.

4. What is the least common multiple of 15 and 20? *Ans.* 60.

5. What is the least number that 3, 5, 8 and 10 will measure? *Ans.* 120.

6. What is the least number that can be divided by the 9 digits separately without a remainder? *Ans.* 2520.

REDUCTION OF VULGAR FRACTIONS.

PROBLEM I.

To reduce a compound Fraction to a simple Fraction.

If $\frac{1}{4}$ of an orange be divided equally among 3 boys, what part of the whole orange will one boy receive?—$\frac{1}{3}$ of $\frac{1}{4}$ is equal to what part of 1?—Thus, $\frac{1}{4}\times\frac{1}{3}=\frac{1}{12}$.

Illustration.—If $\frac{1}{4}$ be divided into three equal parts, it will take 12 such parts to make a whole one; therefore $\frac{1}{3}$ of $\frac{1}{4}$ is $\frac{1}{12}$, *Answer.*

RULE.

Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and they will form the fraction required, which reduce to its lowest terms.

EXAMPLES.

1. How much is $\frac{3}{4}$ of $\frac{5}{8}$?—Thus, $\frac{3}{4}\times\frac{5}{8}=\frac{15}{32}$, *Ans.*

If $\frac{1}{8}$ be divided into 4 equal parts, it will take 32 such parts to make a whole one. Hence, $\frac{1}{4}$ of $\frac{1}{8}$ is $\frac{1}{32}$; then $\frac{1}{4}$ of $\frac{5}{8}$ is 5 times as much$=\frac{5}{32}$. Hence, $\frac{1}{4}$ of $\frac{5}{8}$ is $\frac{5}{32}$, and $\frac{3}{4}$ of $\frac{5}{8}$ is 3 times as much, that is $=\frac{15}{32}$, *Ans.*

2. Reduce $\frac{1}{3}$ of $\frac{9}{11}$ to a simple fraction. *Ans.* $\frac{9}{33}=\frac{3}{11}$.
3. Reduce $\frac{5}{8}$ of $\frac{3}{16}$ to a simple fraction. *Ans.* $\frac{15}{128}$.
4. Reduce $\frac{1}{3}$ of $\frac{5}{6}$ of $\frac{11}{12}$ to a simple fraction. *Ans.* $\frac{55}{216}$.
5. Reduce $\frac{6}{13}$ of $\frac{12}{30}$ of $\frac{5}{78}$ to a simple fraction. *Ans.* $\frac{2}{169}$.
6. How much is $\frac{3}{7}$ of $5\frac{1}{3}$?

   Thus, $5\frac{1}{3}=\frac{16}{3}$ and $\frac{16}{3}\times\frac{3}{7}=\frac{48}{21}$, or $2\frac{2}{7}$, *Ans.*
7. How much is $\frac{3}{8}$ of $22\frac{1}{3}$? *Ans.* $8\frac{3}{8}$.

Note. If the denominator of a fraction be equal to the numerator of another in a compound fraction, they may both be dropped, and the continual multiplication of the other members will produce the fraction in lower terms.

8. Reduce $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{7}{12}$ to a simple fraction.

   Thus, $3\times\frac{}{5}\times\frac{7}{12}=\frac{21}{60}=\frac{7}{20}$, *Ans*
9. Reduce $\frac{1}{5}$ of $\frac{5}{6}$ of $\frac{7}{8}$ of $\frac{8}{11}$ of $5\frac{1}{2}$ to a simple fraction. *Ans.* $\frac{77}{132}=\frac{7}{12}$.
10. Reduce $\frac{3}{4}$ of $\frac{4}{9}$ of $12\frac{3}{4}$ to a simple fraction. *Ans.* $\frac{153}{36}=4\frac{1}{4}$.

PROBLEM II.

To reduce fractions of different denominators to a common denominator.

When fractions have their denominators alike, they are said to have a common denominator, and may then be added or subtracted as easily as whole numbers. Thus, $\frac{1}{9}$ and $\frac{2}{9}$ are $\frac{3}{9}$, &c. But when they have different denominators we must reduce them to a common denominator before we can add or subtract them. Thus, reduce $\frac{1}{4}$ and $\frac{1}{3}$ to a common denominator; that is, find how many parts $\frac{1}{4}$ and $\frac{1}{3}$ must be made into, so that the parts shall be of equal size. Now if we divide $\frac{1}{4}$ into 3 equal parts, it will take 12 such parts to make a whole 1; and if we divide $\frac{1}{3}$ into 4 equal parts, it will take 12 such parts to make a whole 1. Hence the common denominator is 12, and $\frac{1}{4}$ of 12 is $\frac{3}{12}$, and $\frac{1}{3}$ of 12 is $\frac{4}{12}$.

RULE I.

Multiply each numerator into all the denominators except its own, for a new numerator; then multiply all the denominators together for a common denominator, and place it under each new numerator, and it will form the fraction required.

EXAMPLES.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to equivalent fractions having a common denominator.

$$\begin{array}{cccc}
2 & 3 & 5 & 3 \\
4 & 3 & 3 & 4 \\
\hline
8 & 9 & 15 & 12 \\
6 & 6 & 4 & 6 \\
\hline
\frac{48}{72} & \frac{54}{72} & \frac{60}{72} & 72 \text{ common denominator.}
\end{array}$$

2. Reduce $\frac{5}{6}$, $\frac{7}{8}$, and $\frac{2}{5}$ to a common denominator.

Ans. $\frac{200}{240}$, $\frac{210}{240}$, $\frac{96}{240}$.

3. Reduce $\frac{2}{9}$, and $\frac{13}{15}$ to a common denominator.

Ans. $\frac{30}{135}$, $\frac{117}{135}$.

4. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{7}{8}$, and $\frac{5}{6}$ to a common denominator.

Ans. $\frac{144}{288}$, $\frac{192}{288}$, $\frac{252}{288}$, $\frac{240}{288}$.

5. Reduce $\frac{2}{3}$, $\frac{1}{6}$, $\frac{5}{9}$ and $\frac{13}{19}$ to a common denominator.

Ans. $\frac{2052}{3078}$, $\frac{513}{3078}$, $\frac{1710}{3078}$, $\frac{2106}{3078}$.

Compound fractions must be reduced to simple fractions before finding the common denominator; and mixed num-

bers reduced to an improper fraction; or, the fractional part of mixed numbers may be reduced to a common denominator, and annexed to the whole numbers.

6. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ and $\frac{2}{3}$ of $\frac{3}{8}$ to a common denominator.

Ans. $\frac{72}{192}$, $\frac{48}{192}$.

7. Reduce $12\frac{7}{8}$, $\frac{5}{9}$ and $\frac{3}{5}$ to a common denominator.

Ans. $12\frac{315}{360}$, $\frac{200}{360}$, $\frac{216}{360}$.

8. Reduce $18\frac{3}{8}$, and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a common denominator. (Reduce the $18\frac{3}{8}$ to an improper fraction first, then reduce the compound fraction to a simple fraction.)

Ans. $\frac{9408}{512}$, $\frac{120}{512}$.

9. Reduce $10\frac{1}{2}$, $12\frac{3}{5}$, and $\frac{2}{3}$ of $\frac{9}{11}$ to a common denominator. *Ans.* $\frac{3465}{330}$, $\frac{4158}{330}$, $\frac{180}{330}$.

The common denominator of fractions is a common multiple, or such a number as can be divided by all the denominators of the given fractions without a remainder; and multiplying all the denominators continually together produces a common multiple of those factors. But it will not always produce their least common multiple; and as the operations are more easily performed by having the fractions always in their lowest terms, and as their least common multiple is their least common denominator, the following Rule is much preferable.

RULE II.

Find the least common multiple of all the denominators (Problem 2, page 166,) which will be the common denominator of the given fractions. Then divide the common denominator by the denominator of each fraction, and multiply the quotient by the numerator, which will give the new numerator of each fraction; and the new numerator written over the common denominator, will express the fractions in their lowest terms.

EXAMPLES.

1. Reduce $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$, to a common denominator.

Operation.

$$\begin{array}{r|lll} 3 & 3 & 4 & 6 \\ \hline 2 & 1 & 4 & 2 \\ \hline & 1 & 2 & 1 \end{array}$$

$3 \times 2 \times 2 = 12$ common denominator.

The denominator being 12ths, it is evident that the numerator must be in proportion, and to increase this, we take $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ of the 12ths, as below

$12 \div 3 \times 1 = 4$ new numerator, written over the $12 = \frac{4}{12}$.
$12 \div 4 \times 3 = 9$ new numerator, written over the $12 = \frac{9}{12}$.
$12 \div 6 \times 5 = 10$ new numerator, written over the $12 = \frac{10}{12}$.

2. Reduce $\frac{2}{3}$, $\frac{4}{5}$, and $\frac{6}{15}$, to their least common denominator. *Ans.* $\frac{10}{15}$ $\frac{12}{15}$ $\frac{6}{15}$.

3. Reduce $\frac{1}{7}$, $\frac{3}{35}$, and $\frac{9}{14}$, to their least common denominator. *Ans.* $\frac{10}{70}$ $\frac{6}{70}$ $\frac{45}{70}$

4. Reduce $\frac{3}{4}$, $\frac{5}{8}$, and $\frac{1}{3}$ of $\frac{5}{6}$, to their least common denominator.. *Ans.* $\frac{54}{72}$ $\frac{45}{72}$ $\frac{20}{72}$.

5. Reduce $\frac{1}{5}$, $\frac{3}{10}$, $\frac{2}{15}$, and $\frac{7}{30}$, to a common denominator by Rule I. then reduce them by Rule II.

Ans. by Rule I, $\frac{4500}{22500}$ $\frac{6750}{22500}$ $\frac{3000}{22500}$ $\frac{5250}{22500}$. Rule II, $\frac{6}{30}$ $\frac{9}{30}$ $\frac{4}{30}$ $\frac{7}{30}$.

PROBLEM III.

To reduce fractions of higher denominations into those of lower denominations

RULE.

Multiply the numerator of the given fraction, by the common parts of its own integer, and under the product, write the denominator; or make a compound fraction, by comparing the given fraction with all the denominations between it and the denomination you would reduce it to; then reduce the compound fraction to a simple one.

EXAMPLES.

1. Reduce $\frac{1}{320}$ of a pound to the fraction of a penny.

Operation.

$$\begin{array}{r} 1 \\ \times 20 \\ \hline 20 \\ 12 \\ \hline \text{numerator } 240 \end{array}$$

then $\frac{240}{320} = \frac{3}{4}$ *Ans.*

Or, by comparing the given fraction with the several denominations, and making a compound fraction, it will stand thus, $\frac{1}{320}$ of $\frac{20}{1}$ of $\frac{12}{1}$, then $\frac{1}{320} \times \frac{20}{1} \times \frac{12}{1} = \frac{240}{320}$, and $\frac{240}{320} = \frac{3}{4}$ of a penny, the answer as before.

2. Reduce $\frac{3}{160}$ of a pound to the fraction of a shilling.

Thus, $\frac{3}{160} \times 20 = \frac{60}{160} = \frac{3}{8}$s. *Ans.*

3. Reduce $\frac{1}{1280}$ of a pound, to the fraction of a farthing. *Ans.* $\frac{3}{4}$.

4. Reduce $\frac{1}{126}$ of a hogshead, to the fraction of a gallon. *Ans.* $\frac{1}{2}$gal.

Reduce $\frac{2}{1680}$ of a guinea, to the fraction of a penny.
Ans. $\frac{2}{5}$d.

6. Reduce $\frac{7}{1920}$ of a pound Troy, to the fraction of a pwt.
Ans. $\frac{7}{8}$pwt.

7. Reduce $\frac{9}{3520}$ of a mile, to the fraction of a rod.
Ans. $\frac{9}{11}$ rod.

8. Reduce $\frac{9}{1064}$ of a cwt. to the fraction of a pound.
Ans. $\frac{18}{19}$lb.

9. Reduce $\frac{1}{5040}$ of a day, to the fraction of a minute.
Ans. $\frac{2}{7}$m.

10. Reduce $\frac{4}{7}$ of a guinea, to the fraction of a pound.
Compounded thus, $\frac{4}{7}$ of $\frac{28}{1}$ of $\frac{1}{20}=\frac{112}{140}=\frac{4}{5}$ *Ans.*

PROBLEM IV.

To reduce fractions of lower denominations, into those of higher denominations.

RULE.

Multiply the denominator of the given fraction, by the common parts of an integer of the required fraction, for a new denominator, over which write the numerator; or make a compound fraction by comparing the given fraction with the denominations between it and the one you would reduce it to, then reduce this compound fraction to a simple one.

EXAMPLES.

1. Reduce $\frac{3}{4}$ of a penny, to the fraction of a pound.

Operation.

denominator 4
12
———
48
20
———
new denominator 960

then $\frac{\text{numerater}}{\text{denominator}}$ $\frac{3}{960}=\frac{1}{320}$£ *Ans.*

Or by comparing the given fraction with the several denominations and making a compound fraction, it will stand thus—
$\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}=\frac{3}{960}$ and $\frac{3}{960}=\frac{1}{320}$£ answer as before.

2. Reduce $\frac{3}{8}$ of a shilling to the fraction of a pound.
Ans. $\frac{3}{160}$.

3. Reduce $\frac{3}{4}$ of a farthing to the fraction of a pound.
Ans. $\frac{1}{1280}$.

4. Reduce $\frac{1}{2}$ gallon to the fraction of a hogshead. *Ans.* $\frac{1}{126}$.

5. Reduce $\frac{3}{5}$ of a penny, to the fraction of a guinea. *Ans.* $\frac{1}{840}$

6. Reduce $\frac{7}{8}$ of a pwt. to the fraction of a pound Troy. *Ans.* $\frac{7}{1920}$.

7. What part of a mile, is $\frac{9}{11}$ of a rod? *Ans.* $\frac{9}{3520}$.

8. Reduce $\frac{18}{19}$ of a pound, to the fraction of a cwt. *Ans.* $\frac{9}{1064}$.

9. Reduce $\frac{2}{7}$ of a minute, to the fraction of a day. *Ans.* $\frac{1}{5040}$.

10. Reduce $\frac{4}{5}$ of a £ to the fraction of a guinea.
Compounded thus, $\frac{4}{5}$ of $\frac{20}{1}$ of $\frac{1}{28}=\frac{80}{140}$ *Ans.* $\frac{4}{7}$.

ADDITION OF VULGAR FRACTIONS.

RULE.

Reduce compound fractions to single ones, and all of them to their least common denominator (Rule 2, page 169) then the sum of the numerators written over the common denominator, will be the sum of the fractions required.

EXAMPLES.

Add together $12\frac{3}{4}$, $9\frac{5}{6}$, and $\frac{7}{8}$ of $\frac{3}{5}$. Thus, $\frac{7}{8}$ of $\frac{3}{5}=\frac{21}{40}$ then $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{21}{40}$, reduced to a common denominator, by Rule 2, Problem II. are equal to $\frac{90}{120}$, $\frac{100}{120}$, $\frac{63}{120}$, and the sum of the numerators $90+100+63=253$, which written over the common denominator, will be $\frac{253}{120}$, which reduced to a mixed number, is equal to $2\frac{13}{120}$, then the whole numbers $12+9+2\frac{13}{120}=23\frac{13}{120}$ *Ans.*

2. What is the whole amount of $7\frac{1}{3}$ yards, $13\frac{5}{7}$ yards, and $8\frac{1}{5}$ yards? *Ans.* $29\frac{26}{105}$ yds.

3. Add together $\frac{4}{5}$, $\frac{3}{7}$, and $\frac{2}{35}$. *Ans.* $1\frac{2}{7}$.

4. Find the sum of $\frac{2}{3}$, $\frac{3}{8}$, and $\frac{5}{6}$. *Ans.* $1\frac{7}{8}$.

5. Add together $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{8}$. *Ans.* $3\frac{1}{8}$.

6. Find the sum of $18\frac{3}{4}$, and $29\frac{5}{8}$. *Ans.* $48\frac{3}{8}$.

Note. In adding mixed numbers that are compounded with other fractions, reduce them first to improper fractions, and all of them to a common denominator, by Rule 2, page 196, then add as before.

7. Find the sum of $\frac{1}{3}$ of 16, and $\frac{1}{2}$ of $\frac{3}{4}$ of 27. Thus $\frac{1}{3}$ of $\frac{16}{1}=\frac{16}{3}$, and $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{27}{1}=\frac{81}{8}$, then $\frac{16}{3}$ and $\frac{81}{8}$ reduced to a common denominator, will become $\frac{128}{24}$ and $\frac{243}{24}$ which added together, make $\frac{371}{24}=15\frac{11}{24}$ *Ans.*

8. Add together $\frac{1}{4}$ of 25, and $\frac{7}{8}$ of $15\frac{3}{4}$. *Ans.* $20\frac{1}{32}$.

9. Find the sum of $\frac{5}{9}$ and $\frac{2}{5}$ of $\frac{5}{9}$ of 13. *Ans.* $3\frac{4}{9}$.

10. Add together $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{7}{8}$ of $\frac{2}{3}$ of $5\frac{1}{4}$. *Ans.* $3\frac{15}{16}$.

Fractions of different integers (as found in money, weight, &c.) must be reduced to those of the same, before adding; or the value of each fraction may be found by Problem VIII. page 101, and then added together.

11. Add $\frac{3}{8}$ of a shilling to $\frac{2}{9}$ of a pound. Thus, $\frac{3}{8}$s. reduced to the fraction of a pound, by Prob. IV. page 171, equal to $\frac{3}{160}$£; then, $\frac{3}{160}$£, and $\frac{2}{9}$£ added together $=\frac{347}{1440}$ of a pound, whose value by Prob. VIII. page 94, = 4s. 9d. $3\frac{1}{3}$qrs. *Ans.*

Or thus, $\frac{3}{8}$s. = 0s. 4d. 2qrs.
and $\frac{2}{9}$£ = 4 5 $1\frac{1}{3}$

Ans. as before 4 9 $3\frac{1}{3}$

12. Add $\frac{5}{6}$ of a lb. Troy to $\frac{3}{8}$ of a pwt.
Ans. 10oz. 0pwt. 9grs.

13. Add together $\frac{1}{6}$ of a hogshead, $\frac{3}{8}$ of a gallon, and $\frac{5}{8}$ of a quart. *Ans.* 11gal. 0qt. $0\frac{1}{4}$pt.

14. Find the sum of $\frac{2}{7}$ of a cwt. and $\frac{5}{13}$ of a pound.
Ans. 1qr. 4lb. 6oz. $2\frac{6}{13}$dr.

15. Add $\frac{5}{9}$ of a mile, and $\frac{15}{16}$ of a yard together.
Ans. 4fur. 98yds. 2ft. $1\frac{3}{4}$in.

16. Add $\frac{1}{4}$ of a week, $\frac{1}{3}$ of a day, $\frac{1}{2}$ of an hour, and $\frac{3}{4}$ of a minute together. *Ans.* 2d. 2h. 30min. 45sec.

SUBTRACTION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as in addition; then the difference of the numerators written over the common denominator, will be the difference of the fractions required.

EXAMPLES.

1. From $\frac{7}{8}$ take $\frac{2}{3}$ of $\frac{3}{4}$. Thus, $\frac{2}{3}$ of $\frac{3}{4}=\frac{6}{12}$, then $\frac{7}{8}$ and $\frac{6}{12}$ reduced to their least common denominator by Rule 2

page 169, become $\frac{21}{24}$ and $\frac{12}{24}$, and the difference between the numerators, 21 and 12, is 9, which written over the common denominator, becomes $\frac{9}{24}=\frac{3}{8}$ *Ans.*

2. From $\frac{3}{4}$ take $\frac{2}{7}$ of $\frac{5}{8}$. Remains $\frac{4}{7}$.
3. From $\frac{15}{16}$ take $\frac{1}{2}$ of $\frac{3}{4}$. Rem. $\frac{9}{16}$.
4. From $4\frac{3}{4}$ take $3\frac{2}{9}$. Rem. $\frac{55}{36}=1\frac{19}{36}$.
5. What is the difference between $\frac{1}{8}$ and $\frac{1}{18}$? *Ans.* $\frac{5}{72}$.
6. From $14\frac{1}{6}$ take $\frac{1}{5}$ of $17\frac{1}{2}$. *Ans.* $10\frac{2}{3}$.
7. From $\frac{49}{50}$ take $\frac{5}{9}$. *Ans.* $\frac{191}{450}$.

Note. To subtract a fraction from a whole number, take the numerator from its denominator, and place the remainder over the denominator, then take 1 from the whole number.

To subtract mixed numbers without reducing them to improper fractions. When the lower fraction is greater than the upper one, subtract its numerator from the common denominator, and to the difference add the upper numerator, then carry 1 to the lower whole number.

8. From 16 subtract $\frac{5}{9}$. Thus, $\frac{5}{9}$ taken from $\frac{9}{9}$ leaves $\frac{4}{9}$, that is, the numerator 5, subtracted from the denominator 9, leaves 4, which placed over the denominator is $\frac{4}{9}$, then 1 taken from 16, leaves 15. *Ans.* $15\frac{4}{9}$.

9. From 25 take $\frac{13}{25}$. *Ans.* $24\frac{12}{25}$.

10. From $29\frac{2}{3}$ take $15\frac{3}{4}$; thus, $\frac{2}{3}$ and $\frac{3}{4}$ reduced to a common denominator.

make $\frac{8}{12}$ and $\frac{9}{12}$
then $29\frac{2}{3}=29\frac{8}{12}$
$15\frac{3}{4}=15\frac{9}{12}$
Ans. $13\frac{11}{12}$

The lower fraction being greater than the upper one, we proceed thus; $\frac{9}{12}$ subtracted from $\frac{12}{12}$ leaves $\frac{3}{12}$ and $\frac{8}{12}$ (the upper fraction) added to $\frac{3}{12}=\frac{11}{12}$ (that is, we subtract the numerator of the lower fraction (9) from its denominator (12) and to the difference add the upper numerator (8) makes $11=\frac{11}{12}$) we then carry 1 to the whole number, and subtract, and the rem. is $13\frac{11}{12}$.

11. What is the difference between $14\frac{5}{6}$ and $19\frac{2}{9}$. *Ans.* $4\frac{7}{18}$.

12. From $36\frac{5}{48}$ take $9\frac{15}{16}$. *Ans.* $26\frac{8}{48}$

MULTIPLICATION OF VULGAR FRACTIONS.

RULE.

Reduce Compound fractions to simple ones, and whole and mixed numbers to improper fractions. Then multiply the numerators together for a new numerator, and the denominators together for a new denominator

EXAMPLES.

1. What is the product of $5\frac{1}{3}$ multiplied by $\frac{1}{3}$ of $\frac{2}{5}$?
thus, $5\frac{1}{3}=\frac{16}{3}$, and $\frac{1}{3}$ of $\frac{2}{5}=\frac{2}{15}$, then $\frac{16}{3}\times\frac{2}{15}=\frac{32}{45}$. *Ans.*
2. Multiply $\frac{4}{9}$ by $\frac{3}{8}$. *Ans.* $\frac{12}{72}=\frac{1}{6}$.
3. Multiply $\frac{17}{21}$ by $\frac{3}{5}$. *Ans.* $\frac{51}{105}$.
4. Multiply $4\frac{1}{2}$ by $\frac{1}{8}$. *Ans.* $\frac{9}{16}$
5. Multiply $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{3}{4}$ of $\frac{4}{5}$. *Ans.* $\frac{1}{5}$.
6. Multiply $\frac{7}{8}$ of 5 by $\frac{3}{5}$ of 8. Product 21.
7. Multiply $7\frac{1}{2}$ by $9\frac{1}{4}$. Product $69\frac{3}{8}$.
8. Multiply $12\frac{3}{5}$ by $\frac{2}{6}$ of 7. Product $29\frac{2}{5}$.

DIVISION OF VULGAR FRACTIONS.

RULE.

Prepare the fractions as before, then invert the divisor and proceed exactly as in multiplication; and the products will be the quotient required.

How many times is $\frac{2}{9}$ contained in $\frac{5}{8}$?

Thus, by the rule we invert the divisor $\frac{2}{9}$, which becomes $\frac{9}{2}$; then $\frac{5}{8}\times\frac{9}{2}=\frac{45}{16}$, or $2\frac{13}{16}$, the answer.

Illustration of the Rule.—Had it been required to divide $\frac{5}{8}$ by 2 a whole number, instead of 2 ninths, it is evident it would have given the quotient $2\frac{1}{2}$ eighths, or $\frac{5}{16}$; but the divisor, being ninths, will be contained in the dividend 9 times were the whole number is contained 1 time; therefore the quotient $\frac{5}{16}$ is 9 times too small, and must be multiplied by 9, the denominator of the dividing fraction, and 9 $\frac{5}{16}=\frac{45}{16}$, or $2\frac{13}{16}$ the answer as before. This process consists only in multiplying the numerator of the divisor into the denominator of the dividend, and the denominator of the divisor into the numerator of the dividend.

EXAMPLES.

1. Divide $\frac{3}{7}$ by $\frac{5}{8}$ Thus, $\frac{3}{7}\times\frac{8}{5}=\frac{24}{35}$, *Ans.*
2. Divide $\frac{17}{21}$ by $\frac{3}{5}$ Quotient $\frac{85}{63}=1\frac{22}{63}$.
3. Divide $\frac{1}{2}$ of $\frac{2}{3}$ by $\frac{2}{3}$ of $\frac{3}{4}$. Quotient $\frac{24}{36}=\frac{2}{3}$.
4. What is the quotient of $7\frac{5}{8}$ divided by $\frac{5}{8}$ of 7? *Ans.* $\frac{488}{280}=1\frac{26}{35}$.
5. Divide $9\frac{1}{5}$ by $\frac{15}{16}$. Quotient $9\frac{61}{75}$.
6. Divide $4\frac{5}{9}$ by $\frac{5}{9}$ of 4. Quotient $2\frac{1}{20}$.
7. Divide $\frac{5}{9}$ of 4 by $4\frac{5}{9}$. Quotient $\frac{20}{41}$.
8. How many times is $\frac{35}{39}$ contained in $\frac{2}{5}$ of 27? *Ans.* $12\frac{6}{175}$.
9. How many times is $\frac{5}{16}$ contained in $\frac{49}{50}$? *Ans.* $3\frac{17}{125}$.
10. Divide 19 by 37. Quotient $\frac{19}{37}$.
11. At $\frac{4}{9}$ of a dollar per bushel, how many bushels of oats can be bought for \$$19\frac{5}{6}$, or \$$\frac{119}{6}$? *Ans.* $44\frac{15}{24}$bu.
12. At $\frac{7}{8}$ of a dollar per bushel, how much corn can be bought for $27\frac{2}{5}$ dollars? *Ans.* $31\frac{11}{35}$ bushels.

RULE OF THREE DIRECT IN VULGAR FRACTIONS.

RULE.

Prepare the fractions as in multiplication: then state the question in the same manner as taught in the Rule of Three in whole numbers; then invert* the first term and multiply all three of the terms continually together, and the product will be the answer in the same name of the second or middle term.

EXAMPLES.

1. If $\frac{1}{8}$ of a barrel of flour cost $\frac{5}{6}$ of a dollar, what will $\frac{3}{4}$ of a barrel cost?

Thus, $\overset{\text{bbl.}}{\frac{1}{8}} : \overset{\$}{\frac{5}{6}} :: \overset{\text{bbl.}}{\frac{3}{4}}$. Then the divisor or first term being inverted will stand thus $\frac{8}{1}\times\frac{5}{6}\times\frac{3}{4}=\$\frac{120}{24}$, and $\$\frac{120}{24}=$ 5dollars, the answer.

The same, by analysis.—If $\frac{1}{8}$ cost $\frac{5}{6}$ of a dollar 1 barrel,

* The reason of inverting the first term in this rule is very evident, since fractions depend on the same principle as whole numbers, and this produces the same effect as that of multiplying the second and third terms together and dividing by the first; therefore, the first term being a divisor, we invert it as taught in Division of Vulgar Fractions

$\frac{8}{8}$ will cost 8 times as much. Thus, 8 times $\frac{5}{6}=\frac{40}{6}$; then $\frac{3}{4}$ of $\$\frac{40}{6}=\$\frac{120}{24}$, or 5 dollars, the answer as before.

2. If $\frac{2}{3}$ of a bushel of wheat cost $\frac{7}{8}$ of a dollar, what will $8\frac{1}{5}$ bushels cost?

Thus, $\frac{2}{3} : \frac{7}{8} :: \frac{41}{5}$, and $\frac{3}{2}\times\frac{7}{8}\times\frac{41}{5}=\frac{861}{80}=\$10\frac{61}{80}$, *Ans.*

3. If $10\frac{61}{80}$ dollars will buy $8\frac{1}{5}$ bushels of wheat, how much wheat will $\frac{7}{8}$ of a dollar buy?

Thus, $10\frac{61}{80}=\frac{861}{80}$, and $8\frac{1}{5}=\frac{41}{5}$; then $\overset{\$}{\frac{861}{80}} : \overset{\text{bu.}}{\frac{41}{5}} :: \overset{\$}{\frac{7}{8}} : \overset{\text{bu.}}{\frac{2}{3}}$, *Ans.*

4. If $\frac{5}{8}$ of a yard cost $\frac{4}{7}$ of a pound, what will $\frac{9}{15}$ of an Ell English come to?

$\frac{5}{8}$yd. $=\frac{5}{8}$ of $\frac{4}{1}$ of $\frac{1}{5}=\frac{20}{40}$, or $\frac{1}{2}$ Ell English.

Then, as $\overset{\text{Ell.}}{\frac{1}{2}} : \overset{£}{\frac{4}{7}} :: \overset{\text{Ell.}}{\frac{9}{15}}$; and $\frac{2}{1}\times\frac{4}{7}\times\frac{9}{15}\times\overset{£}{\frac{72}{105}}=\overset{\text{s.}}{13}\ \overset{\text{d.}}{8}\ \overset{\text{qrs.}}{2\frac{6}{21}}$.

5. If $\frac{3}{5}$ of a cwt. cost $\frac{7}{9}$ of a dollar, what will $15\frac{2}{3}$cwt. come to? *Ans.* \$20, 30c. $8\frac{6}{10}$m.+

6. If $\frac{1}{3}$ of $\frac{2}{5}$ of an acre of land cost \$9, what will $30\frac{1}{4}$ acres come to? *Ans.* \$2041,$87\frac{1}{2}$.

7. If $\frac{9}{16}$ of a vessel cost 1236 dollars, what are $\frac{3}{32}$ of her worth? *Ans.* \$206.

8. A merchant sold $5\frac{1}{2}$ pieces of cloth, each containing $12\frac{2}{3}$ yards, at 9s. $\frac{1}{2}$d. per yard; what did the whole come to? *Ans*, £31 9s. 10d. $3\frac{1}{3}$qrs.

9. At $\$3\frac{5}{8}$ per cwt., what will $9\frac{2}{3}$lbs. come to?

Ans. $\$\frac{841}{2688}$, or 31cts. $2\frac{8}{10}$m.+

10. A person owning $\frac{4}{5}$ of a vessel sold $\frac{2}{3}$ of his share for 835 dollars; what was the whole vessel worth at that rate? *Ans.* 1565 dollars $62\frac{1}{2}$ cents.

RULE OF THREE INVERSE IN VULGAR FRACTIONS.

RULE.

Prepare the given fractions, and state the question as in direct proportion; then invert the third term and multiply all three of the terms together, the product will be the answer in the same name of the middle term.

EXAMPLES.

1. How much flannel that is $\frac{3}{5}$ of a yard wide, will line $5\frac{3}{4}$ yards of cloth which is $1\frac{1}{2}$ yards wide?

As $\overset{\text{yds. wide.}}{1\frac{1}{2}} : \overset{\text{yds.}}{5\frac{3}{4}} :: \frac{3}{5}$; and $\frac{3}{2}\times\frac{23}{4}\times\frac{5}{3}=\frac{345}{24}=\overset{\text{yards.}}{14\frac{3}{8}}$ Answer.

2. If a man can do a piece of work in $9\frac{2}{3}$ days, when the days are $12\frac{1}{2}$ hours long, in how many days will he do the same when the days are but $9\frac{1}{3}$ hours long? *Ans.* $12\frac{53}{56}$ days.

3. If $5\frac{7}{8}$ yards of cloth which is $2\frac{1}{8}$ yards wide will make a cloak, how many yards of cloth that is only $\frac{3}{4}$yd. wide will make one? *Ans.* $16\frac{31}{48}$ yards.

4. If 14 men can do a piece of work in $13\frac{1}{6}$ days, how many men will do the same in $6\frac{7}{12}$ days? *Ans.* 28 men.

5. If I lend my friend 250 dollars for $\frac{3}{8}$ of a year, how much ought he to lend me $\frac{5}{8}$ of a year to requite the favour? *Ans.* \$150.

6. How much baize that is $1\frac{1}{4}$yds. wide, will line $16\frac{3}{5}$ yards of camblet that is $\frac{3}{4}$yd. wide? *Ans.* $9\frac{24}{25}$

7. How much length that is $8\frac{1}{5}$ inches wide, will make a square foot? *Ans.* $17\frac{23}{41}$

8. If $25\frac{2}{7}$s. will pay for the carriage of a cwt. $145\frac{1}{4}$ miles, how far may $6\frac{1}{2}$ hundred weight be carried for the same money? *Ans.* $22\frac{9}{26}$ miles

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COMPOUND PROPORTION, OR THE DOUBLE RULE OF THREE,

Teaches to solve by one statement such questions as would require two or more statements in simple proportion whether direct or inverse,—having (commonly) five terms given to find a sixth, the first three being a supposition, and the last two a demand.

RULE.

State the question, that is, place the terms of supposition so that the one which is the principal cause of loss, gain or action, stand in the first place; that which denotes the space of time, distance of place, &c. in the second place, and the remaining term of supposition in the third place. Place the other two terms under those of the same kind in the supposition.

If the blank place or term sought fall under the third place, the proportion is direct; then multiply the first and second terms together for a divisor, and the other three for a dividend.

But if the blank place fall under the first or second term, the proportion is inverse; then multiply the third and fourth terms together for a divisor, and the other three for a dividend, and the quotient will be the answer.

EXAMPLES.

1. If 7 men in 16 days reap 84 acres of grain, how many acres can 40 men reap in 5 days?

```
       men. days. acres.  [position.
         7 : 16 :: 84 terms of sup-
        40 :  5 ::    terms of de-
              84              [mand
             420
              40
             ------- acres.
7×16=112)16800(150 Ans.
         112
          560
          560
```

In this example, the blank place falls under the 3d term; therefore the proportion is direct; and we multiply the first and 2d terms, viz.: $7\times16=112$, for a divisor, and the other three, viz: $84\times40\times5$ together for a dividend $=16800$; and $16800\div112=150$, *Ans.*

By analysis.—If 7 men can reap 84 acres in 16 days, 1 man can reap $\frac{1}{7}$ of $84=12$ acres in 16 days, which is $\frac{12}{16}=\frac{3}{4}$ of an acre a day; and 40 men will reap 40 times $\frac{3}{4}=\frac{120}{4}=30$ acres in 1 day, and in 5 days they will reap 5 times $30=150$ acres; the answer as before.

By two statements in simple proportion.—First; taking into consideration the number of men employed, we have the following proportion;—if 7 men in a certain number of days, reap 84 acres, how many acres will 40 men reap in the same time?—As 7 : 84 :: 40 ...... which gives for the fourth term 480 acres. Then, taking into consideration the number of days, it will stand thus: if a certain number of men in 16 days, reap 480 acres, how many acres will they reap in 5 days?

	days.		acres.		days.		acres.	
As	16	:	480	::	5	:	150	*Ans.*

2. If 100 dollars in 12 months gain 6 dollars interest, what principal in 7 months will gain 14 dollars interest?

$	mo.		$	
100 :	12 : :		6	
	7 : :		14	

Ans. $400.

Here the blank place falls under the first term; therefore the proportion is inverse, and we multiply the third and fourth terms together ($6\times7=42$) for a divisor, and the other three ($100\times12\times14=16800$) for a dividend; and $16800\div42=400$, *Answer*.

3. If 100 dollars gain 6 dollars in a year, what will 400 dollars gain in 7 months?

$		mo.		$
100	:	12	: :	6
400	:	7	: :	

Ans. 14 dollars.

4. If 100 dollars in 1 year gain 6 dollars, in what time will 400 dollars gain 14 dollars?

$		mo.		$
100	:	12	: :	6
400	:		: :	14

Ans. 7 months.

5. If $247,50 will pay the board of 15 persons eleven weeks, how much will it cost to board 9 persons 5 weeks?

15	:	11	: :	247,50
9	:	5	: :	

Ans. $67,50.

6. If 5 men build 150 rods of wall in 12 days, how many rods will 14 men build in 9 days? *Ans.* 315 rods.

7. If 5 men build 150 rods of wall in 12 days, in how many days will 14 men build 315 rods? *Ans.* 9 days.

8. If 5 men build 150 rods of wall in 12 days, how many men will build 315 rods in 9 days? *Ans.* 14

9. A usurer put out 125 dollars at interest, and when the same had been out 9 months he received for principal and interest $132,50; at what rate per cent did he receive interest? *Ans.* 8 per cent.

10. If the carriage of $6\frac{1}{4}$cwt. 140 miles cost $28,50, what will be the cost of carrying 8cwt. 3qrs., 64 miles at the same rate? *Ans.* $18,24.

11. If a footman travel 240 miles in 12 days, when the days are 12 hours long, in how many days will he travel 720 miles when the days are 16 hours long? *Ans.* 27d.

12. If 30 men in 20 days build 300 rods of wall, how many men will build four times as much in a fifth part of the time? *Ans.* 600.

13. If 6 men build a wall 20 feet long 6 feet high and 4

feet thick in 16 days, in what time will 24 men build one 200 feet long 8 feet high and 6 feet thick?

men.		da.		ft. long. ft. h. ft. thick.
6	:	16	::	$20 \times 6 \times 4$
24	:		::	$200 \times 8 \times 6$ *Ans.* 80 days.

Questions.

1. What is the common measure or divisor of two or more numbers?
2. What is the common multiple of two or more numbers?
3. When two or more fractions have the same denominator, what is it called?
4. How do you find the greatest common divisor of two numbers?
5. How do you find the least common multiple of two or more numbers?
6. How do you reduce a compound fraction to a simple one?
7. How do you reduce fractions having different denominators, to a common denominator?—To their least common denominator?
8. How do you reduce fractions of higher into lower denominations?
9. How do you reduce fractions of lower into higher denominations?
10. How do you add Vulgar Fractions?
11. How do you subtract Vulgar Fractions?
12. How do you multiply Vulgar Fractions?
13. How do you divide Vulgar Fractions?
14. How do you perform the Rule of Three Direct in Vulgar Fractions?
15. How do you perform the Rule of Three Inverse in Vulgar Fractions?
16. What is Compound Proportion, or the Double Rule of Three?
17. What is the Rule for Compound Proportion?

INVOLUTION.

Involution, or the raising of powers, is the multiplying any given number into itself, and that product by the former multiplier, and so on; and the products thus produced are called the powers of the given number.

The first power or root of any number is that number itself. If the first power be multiplied by itself, it produces the second power or square. If the square be multiplied by the first power, it produces the third power, or cube, &c.

The number denoting the height of the power is called the index, or exponent of that power. Thus, 9^4 denotes that 9 is raised to its 4th power.

EXAMPLES.

1. What is the 4th power of 9?

9 the first power or root.
9
81 = 2d power, or square $= 9^2$.
9
729 = 3d power, or cube $= 9^3$.
9
6561 = 4th power, or biquadrate $= 9^4$.

2. What is the square of 6? *Ans.* 36.
3. What is the square of 12? *Ans.* 144.
4. What is the cube, or 3d power of 6? *Ans.* 216.
5. What is the cube of 12? *Ans.* 1728.
6. What is the square of ,085? *Ans.* ,007225.
7. What is the cube of 12,5? *Ans.* 1953,125.
8. What is the biquadrate of 16? *Ans.* 65536.
9. What is the square of $6\frac{1}{4}$?

Thus, $6\frac{1}{4}=\frac{25}{4}$, and $\frac{25}{4}\times\frac{25}{4}=\frac{625}{16}=39\frac{1}{16}$, *Ans.*

10. What is the cube of $\frac{7}{8}$? *Ans.* $\frac{343}{512}$.
11. What is the square of $3\frac{1}{3}$? *Ans.* $11\frac{1}{9}$.
12. What is the cube of $3\frac{1}{3}$? *Ans.* $37\frac{1}{27}$.
13. What is the cube of $6\frac{1}{4}$? *Ans.* $244\frac{9}{64}$.

EVOLUTION, OR EXTRACTION OF ROOTS,

Is the method of finding the root of any given power, or number.

The Root, as we have seen, is that number which, being multiplied continually into itself, produces the given power.

The Square Root is a number which, being multiplied by itself or squared, will produce the given number. Thus, the square root of 64 is 8, because 8^2, that is, $8\times8=64$; and the cube root of 512 is 8, because 8^3, that is, $8\times8\times8=512$; and so for any other number or power.

Although there is no number that will not produce a perfect power by involution, yet there are many numbers of which precise roots cannot be obtained; but by the help of decimals, we can approximate towards the root to any assigned degree of exactness.

Numbers whose precise roots can be obtained are called rational numbers; but those whose precise roots cannot be determined are called surd numbers,

This character ($\sqrt{}$) prefixed to any number shows that the square root of that number is to be extracted. Thus, $\sqrt{16}$ shows that the square root of 16 is to be extracted. Other roots are denoted by the same character, with the index of the required root placed before it. Thus, $\sqrt[3]{27}$

denotes that the cube root of 27 is to be extracted; and $\sqrt[4]{81}$ = 4th root of 81.

When the power is expressed by several numbers, with the signs + or — between them, a line or vinculum is drawn from the top of the sign over all the parts of it. Thus, the square root of 30—5 is expressed $\sqrt{30-5}$.

A TABLE OF THE SQUARES AND CUBES OF THE NINE DIGITS.

Roots.	1	2	3	4	5	6	7	8	9
Squares.	1	4	9	16	25	36	49	64	81
Cubes.	1	8	27	64	125	216	343	512	729

EXTRACTION OF THE SQUARE ROOT.

To extract the Square Root of any number, is, to find a number which, being multiplied into itself, shall produce the given number.

RULE.

1. Point off the given number into periods of two figures each, by putting a dot over the units, another over the place of hundreds, and so on; and if there be decimals, point them in the same manner from units towards the right hand, which dots will show the number of figures the root will consist of.

2. Find the greatest square number in the first, or left hand period; place its root as a quotient in division, and place the square number under the period and subtract it therefrom; and to the remainder, bring down the next period for a dividend.

3. Double the root already found, and place it at the left hand of the dividend for a divisor.

4. Place such a figure at the right hand of the divisor, and also the same figure in the root, as, when multiplied into the divisor thus increased, the product shall be equal to, or next less than the dividend; this will be the second figure in the root.

5. Multiply the whole increased divisor by the last figure of the root; place the product under the dividend.

subtract it therefrom, and to the remainder bring down the next period for a new dividend.

6. Double the figures already found in the root for a new divisor; and from these find the next figure in the root as last directed; and continue the operation in the same manner until all the periods are brought down.

Note. When there is a deficiency in any period of decimals, you may annex a cipher; or, when there is a remainder, you may continue the operation to decimals, by annexing periods of ciphers.

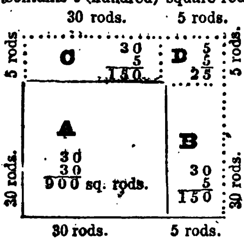
EXAMPLES.

1. What is the length of one side of a square field which contains 1225 square rods? or what is the square root of 1225?

Operation

```
     1225(35
     9
    ----
  65)325
     325
     ---
      00
```

Illustration.—By the Rule, we point the given number into periods of two figures each, by putting a dot over the unit's place and another over the place of hundreds, making 2 periods, which show that the root will consist of 2 places of figures, viz.: a ten and a unit. This Rule for determining the number of figures of which the root will consist is founded on the known principle that the places of figures in the product of any two factors cannot exceed the number of places contained in both of those factors, nor can they be but one less than the places in both factors: and as the square root of any power is a factor which, being multiplied into itself, exactly produces that power, consequently, any square number contains just twice as many places of figures as its root, or at least, but one less than twice that number. Then the first period being 12 (hundreds,) we seek for the greatest square number that is contained therein, which we find to be 9 (hundreds,) the root of which is 3 (tens=30.) We therefore place 3 (tens) in the root, and its square 9 (hundreds,) under the period 12 (hundreds,) which being deducted, the remainder is 3 (hundreds,) to which we join the next period, 25, making the dividend 325. These 3 (tens=30) in the quotient, or root, it will be recollected, are the length of one side of a square field which contains 9 (hundred) square rods, which are contained in the square figure A; (30×30=900,) and 900 square rods deducted from 1225 square rods leaves 325 square rods to be added to the square figure A.



Now to dispose of the remaining 325 rods so as to retain the square form of the figure A, it is evident that we must make the addition on two sides: the length of each is 30 rods, and 30+30=60 rods, or which is the same thing, double the root already found: then, 3 (tens) makes 6 (tens,) or 60 for a divisor; then if we divide 325 by 60, or, which is the same thing, neglect the cipher in the divisor and divide 6 (tens) into 32 (tens,) it shows that the breadth of the addition must be 5 rods, which is the next figure in the root; then if we examine the figure, we shall find that it is not yet complete, for there yet remains a small square in the corner D, each side of which is 5 rods,= to the last quotient figure. This quotient figure, 5, we must add to the divisor 60, (by the Rule) making 65 the whole divisor. Or, which is the same thing, we place the figure 5 at the right

hand of the 6 (tens,) making 65 the whole divisor, which multiplied by the quotient figure 5 gives the whole number of square rods in the addition around the sides of the square, A: 65 × 5=325 square rods, which being deducted from the dividend, 325, leaves 00.

Hence we find that the square root of 1225 is 35, which is the length of one side of the field. 2

Proof.—This question may be proved by Involution; thus, 35, or 35 × 35=1225: or by adding the several parts of the figure together;

Thus, A contains 900 square rods.
B " 150 "
C " 150 "
D " 25 "
1225 square rods as before.

2. What is the square root of 55696?

Operation.

```
 55696(236 Ans.            Proof.
 4                           236
43)156                       236
   129                      1416
466)2796                    708
    2796                   472
      00                   55696
```

3. What is the square root of 6440,0625?

```
 6440,0625(80,25 Ans.        Proof
 64                          80,25
1602)40,06                   80,25
     32,04                   40125
16045)80225                 16050
      80225                64200
        000               6440,0625
```

4. What is the square root of 138384? *Ans.* 372
5. What is the square root of 1648,36? *Ans.* 40,6.
6. What is the square root of 6625476? *Ans.* 2574.
7. What is the square root of 488,631025? *Ans.* 22,105.
8. What is the square root of ,002304? *Ans.* ,048.
9. What is the square root of 30138696025?

Ans. 173605.

10. What is the square root of 1355?—In this example, after bringing down all the figures there is a remainder, to which we annex a period of ciphers and continue the operation to decimals; and there is still a remainder, to

which we annex another period of ciphers, and proceed as before.—And in this way we may continue the operation to any assigned degree of exactness. But we can never obtain the precise root, for the right hand figure in the dividend will always be a cipher; but the last figure in each divisor is the same as the last figure in the root, and no one of the nine digits, multiplied into itself, will produce a number ending with a cipher. Consequently, there will always be a remainder, whatever may be the quotient figure. *Ans.* 36,81+.

11. What is the square root of 45? *Ans.* 6,708+.
12. What is the square root of 2972? *Ans.* 54,516+.
13. What is the square root of 1546,8? *Ans.* 39,3293+.

TO EXTRACT THE SQUARE ROOT OF VULGAR FRACTIONS.

RULE.

Reduce the fraction to its lowest terms, and extract the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

If it be a mixed number, reduce it to an improper fraction; or if it be a surd, reduce it to a decimal, and then extract its root.

14. What is the square root of $\frac{147}{192}$?—Thus, $\frac{147}{192}=\frac{49}{64}$; then $\sqrt{49}=7$, the numerator, and $\sqrt{64}=8$, the denominator. *Ans.* $\frac{7}{8}$.
15. What is the square root of $\frac{125}{405}$? *Ans.* $\frac{5}{9}$.
16. What is the square root of $\frac{225}{1024}$? *Ans.* $\frac{15}{32}$.
17. What is the square root of $18\frac{7}{9}$? *Ans.* $4\frac{1}{3}$.
18. What is the square root of $162\frac{9}{16}$? *Ans.* $12\frac{3}{4}$.

SURDS.

19. What is the square root of $\frac{15}{20}$? *Ans.* ,866+.
20. What is the square root of $\frac{11}{12}$? *Ans.* ,9574+.

APPLICATION AND USE OF THE SQUARE ROOT.

1. A certain square pavement contains 25600 square stones of equal size; how many are contained in one of its sides? $\sqrt{25600}=160$, *Ans.*

2. A General has an army of 5625 men; how many must be placed in rank and file to form them into a square? $\sqrt{5625}=75$, *Ans.*

3. How many trees in each of the rows of a square orchard that contains 5184? $\sqrt{5184}=72$, *Ans.*

4. There is a circle whose area, or superficial contents, is 2025 feet; what is the length of the side of a square containing an equal number of feet? *Ans.* 45 feet.

PROBLEM II.—The square root of the product of any two numbers is a mean proportional between those numbers.

1. What is the mean proportional between 19 and 76? $\sqrt{76\times19}=38$, *Ans.*

2. A certain field is 84 rods long and 21 rods wide; what is the length of a square field which shall contain an equal number of rods? $\sqrt{84\times21}=42$ rods, *Ans.*

PROB. III.—The length of the side of a square being given to make another square which shall contain 2, 3, 4, &c. times as much.

RULE.

Multiply the square of the given side by the given proportion, and extract the square root of the product.

1. The length of the side of a certain square garden is 12 rods; I demand the length of the side of another garden containing 4 times as much? $\sqrt{12\times12\times4}=24$rds. *Ans.*

2. If the side of a square be 5 feet, what is the length of the side of another square which will contain 25 times as much? $\sqrt{5\times5\times25}=25$ feet, *Ans.*

PROB. IV.—To place any number of soldiers so that the number in rank shall be double, triple, &c. the number in file, (or, to make a figure of any given number, so that the length shall be double, triple, &c. the breadth.)

RULE.

Extract the square root of $\frac{1}{2}$, $\frac{1}{3}$, &c., of the given number, which will be the number in file, or (breadth;) double, triple, &c. the number in file, (or breadth,) and that will be the number in rank, (or length.)

1. Let 6498 men be so formed that the number in rank may be double the number in file.

$6498 \div 2 = 3249$, and $\sqrt{3249} = 57$ in file; and

$57 \times 2 = 114$ in rank, *Ans.*

2. A man wishes to plant 1875 trees in an orchard which is 3 times as long as it is broad, so that they may stand in rows in equal distances apart; how many rows must he make, and how many trees must he plant in each row? *Ans.* 25 rows of 75 trees.

3. There is a certain lot of land containing 6 acres 2 roods and 18 rods in the form of a long square, the length of which is twice as much as the breadth; required the length and the breadth.

Ans. The width is 23 rods, and the length 46 rods.

PROB. V.—The diameter of a circle being given, to make another circle which shall be proportionably greater or smaller than the given circle.

Note. The areas, or contents of circles, are in proportion to the squares of their diameters or circumferences.

RULE.

Square the given diameter; then multiply by the given proportion, if greater, (but divide if smaller,) and extract the square root of the product, (or quotient,) which will give the required diameter.

1. Suppose there is a certain garden whose diameter 9 rods, and it is required to lay out another which shall contain 3 times as much; what must be its diameter?

$\sqrt{9 \times 9 \times 3} = 15,58+$ rods, *Ans.*

2. There is a circular field whose area, or contents, is 64 rods, the diameter of which is 9 rods; required the diameter of another which shall contain just one-fourth as much. *Ans.* 4,5rds.

3. The quantity of water discharged through a certain pipe, which is 2,5 inches in diameter, will fill a certain cistern in one hour; what is the diameter of another pipe which will fill another cistern four times as large, in the same time? $\sqrt{2,5 \times 2,5 \times 4} = 5$ inches diameter, *Ans.*

PROB. VI.—The sum of two numbers and their product given, to find those numbers.

RULE.

From the square of their sum subtract 4 times their product, and the square root of the remainder will be their difference; half the said difference added to the half sum will be the greater of the two numbers; and half the said difference subtracted from the half, will be the lesser number.

1. The sum of two numbers is 58, and their product is 792; what are those numbers?

$58 \times 58 = 3364 =$ square of their sum.

$792 \times 4 = 3168 = 4$ times their product.

$\sqrt{196} = 14$, difference of the numbers.

$\frac{1}{2}$ of $58 =$ 29, the half sum.

$\frac{1}{2}$ of $14 = + -$ 7, the half difference.

36, the greater number.

22, the lesser number.

PROB. VII.—The difference and the product of two numbers given, to find those numbers.

RULE.

Add the square of half the difference of the numbers to their product, and the square root of that amount will be half the sum of the two numbers. Then to the half sum add half the difference, gives the greater number; and from the half sum, subtract half the difference, gives the lesser number.

EXAMPLE.

1. The difference of two numbers is 9, and their product is 442; what are those numbers?

Difference of the numbers	9	Product 442	
		+ 20,25	
Half difference,	4,5		[sum.
	4,5	$\sqrt{462,25} = 21,5$, half	
	225	Then $\frac{1}{2}$ their diff.	4,5
	180	Add gives the greater	26,0
Square of half diff.	20,25	Subtract gives the less	17,0

Questions.

1. What is Involution?
2. What is Evolution?
3. What is the Root of any power?
4. What is the Square Root?
5. Can the precise roots of all powers be found?
6. What are rational numbers?
7. What are surd numbers?
8. What is it to extract the square root?
9. What is the Rule?
10. When there is a remainder, how may the operation be continued?
11. How do you extract the square root of a Vulgar Fraction?

EXTRACTION OF THE CUBE ROOT.

We have seen that any number multiplied into itself produces a square, and that the square multiplied again by that number produces a cube; and likewise, that the number itself is the root of the given cube.

Hence, to extract the cube root of any given number is, to find a number which, being raised to its third power, that is, multiplied into its square, shall produce the given number.

A solid body having six equal sides, and each of the sides an exact square, is a cube; and since the length, breadth and thickness, are the same, it is evident that the length of one side of the given body is the cube root of that body; for, the length, multiplied by the breadth, multiplied by the thickness, will give the cubic contents, &c.

Thus, the cubic contents of a square block a foot long, a foot wide and a foot thick is $1\times1\times1=1$ foot. The cubic feet contained in a block 2 feet long, 2 feet wide and 2 feet thick, is $2\times2\times2=8$ cubic feet. Hence the cube root of 8 is 2, because 2^3, that is, $2\times2\times2=8$.

RULE.

I. Separate the given number into periods of three figures each, by putting a point over the unit figure, and every third figure from the place of units, towards the left, and if there be decimals, point them from the unit's place towards the right in the same manner.

II. Find the greatest cube in the left hand period, and put its root in the quotient.

III. Subtract the cube thus found from the said period, and to the remainder bring down the next period, calling this the dividend.

IV. Multiply the square of the quotient by 300, calling it the divisor.

V. Seek how often the divisor may be had in the dividend, and place the result in the quotient, (root.)

VI. Multiply the divisor by this last quotient figure, and place the product under the dividend.

VII. Multiply the former quotient figure, or figures, by the square of the last quotient figure, and that product by 30, and write the product under the last; then place the cube of the last quotient figure under these two products, and call their amount the subtrahend.

VIII. Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend, with which proceed as before; and so on till the whole is finished.

Note 1. If the subtrahend happens to be larger than the dividend, the last quotient figure must be made one less, and a new subtrahend found.

2. When it happens that the divisor is not contained in the dividend, we must put a cipher in the quotient, and bring down the next period for a new dividend; and multiply the square of the whole quotient by 300 for a new divisor.

3. When there is a remainder after bringing down all the periods, we may annex periods of ciphers, and continue the operation to decimals.

EXAMPLES.

1. What is the length of the side of a cubic block, which contains 12167 solid or cubic inches?

Operation.*

$$
\begin{array}{rl}
 & \dot{1}2\dot{1}67(23 \\
 & 8 \\
\hline
2\times2\times300=1200) & 4167 \text{ dividend.} \\
\hline
 & 3600 \\
2\times3\times3\times30= & 540 \\
3\times3\times3= & 27 \\
\hline
 & 4167 \text{ subtrahend.} \\
\hline
 & 00
\end{array}
$$

* We have seen that the square of any number contains just twice as many figures as the number itself, or at least, but one less than twice that number. So also the cube (being a number multiplied into its square) contains just 3 times

2. What is the cube root of 34328,125?

```
                         34328,125(32,5
                         27
      3×3×300=2700)7328 dividend.
                         5400
     3×2×2×30=           360
     2×2×2=                8
                         5768 subtrahend.
32×32×300=307200)1560125 dividend.
                         1536000
    32×5×5×30=             24000
    5×5×5=                   125
                         1560125 subtrahend.
                            0000
```

---

as many places of figures as the number itself, or at least, but two figures less than 3 times that number. Hence, by pointing any number into periods of 3 figures each, as directed in the Rule, we may at once find how many figures the root will consist of.—Pointing the above example, we have two periods: hence we find that the root will consist of two figures, viz.: a figure of tens and a figure of units.

We then seek for the greatest cube in the first or left hand period, 12 [thousands;] this we find to be 8 [thousands,] the root of which is 2 [tens:] placing the 2 [tens] for the first figure in the root, and its cube, 8 [thousands,] under the 12 [thousands, and subtracting it therefrom, the remainder is 4 [thousands,] to which we bring down the next period, 167, making 4167 inches, which remain.

We have now disposed of 8000 inches in a cube, the length of each side of which being 2 [tens]=20 inches, and $20\times20\times20=8000$. Now suppose we make a cubic block, and suppose each side of it to be 2 [tens]=20 inches, it will contain 8000 cubic inches. We must now enlarge this block by the addition of 4167 cubic inches, so that the block shall retain its cubic form; and in order to do this it is plain that we must make the addition on three sides of it. Now the square contents of each of these sides is $20\times20=400$, and 400×3, the number of sides on which the addition is to be made, gives 1200, the square contents in the given sides. (But we may obtain the square contents in these sides by neglecting the cipher in the 2 tens=20, and multiply the square of this quotient figure 2 by 300, and it will produce the same. Thus, $2\times2\times300=1200$ as above; and thus the rule is formed.)

Now it is evident that the 4167 inches, which are to be added to this block divided by 1200, the square inches contained in the sides on which the additions are to be made, will show the thickness of the additions to be made on each of the three sides. Thus, 1200 is contained in 4167, 3 times, which shows that the thickness of the additions must be 3 inches. We place the 3 in the root, and multiply the square contents, 1200, by the thickness, 3 inches: that is, the last quotient figure; making 3600 cubic inches contained in these additions, which we place under the dividend. Now after these additions are made to the cube, there are 3 vacancies on the corners, each of which is 3 inches wide, and 3 inches thick, and 20 inches long, containing $3\times3\times20=180$ cubic inches. This, multiplied by 3, gives the whole number of inches in the three vacancies, =540 cubic inches. But by the rule we neglect the cipher, and multiply the former quotient figure, 2 [tens,] by the square of the last, and that product by 30, which produces the same effect. Thus, $2\times3\times3\times30=540$, which we place under the former. Now if we

3. What is the cube root of 21952? *Ans.* 28.
4. What is the cube root of 250047? *Ans.* 63.
5. What is the cube root of 614125? *Ans.* 85.
6. What is the cube root of 21024576? *Ans.* 276.
7. What is the cube root of 94818,816? *Ans.* 45,6.
8. What is the cube root of 7612,812161? *Ans.* 19,67+
9. What is the cube root of ,121861281? *Ans.* ,495×
10. What is the cube root of ,000021952? *Ans.* ,028.
11. What is the cube root of $\frac{8}{27}$?

Thus, $\sqrt[3]{8}=2$, the numerator, and $\sqrt[3]{27}=3$, the denominator. *Ans.* $\frac{2}{3}$.

12. What is the cube root of $\frac{343}{512}$? *Ans.* $\frac{7}{8}$.
13. What is the cube root of $\frac{1}{216}$? *Ans.* $\frac{1}{6}$.
14. What is the cube root of $\frac{512}{729}$? *Ans.* $\frac{8}{9}$.
15. What is the cube root of $\frac{2197}{4096}$. *Ans.* $\frac{13}{16}$.

APPLICATION OF THE CUBE ROOT.

1. How many solid or cubic feet are contained in a cubical block which is 8 feet long, 6 feet wide and 4 feet thick? $8\times6\times4=192$, *Ans.*

2. How many solid or cubic feet of earth were thrown out of a cellar which is 32 feet long, 25 feet wide, and 10 feet deep? *Ans.* 8000.

3. What is the length of the side of a cube which contains 8000 solid or cubic feet? $\sqrt[3]{8000}=20$ft. *Ans*

4. A bushel contains 2150,420 solid or cubic inches; what is the length of the side of a cubic box which shall contain that quantity? *Ans.* 12,9+inches.

5. The side of a certain cubical box measures 1 foot; what is the length of the side of another that is 8 times as large? $1\times1\times1=1\times8=8$, and $\sqrt[3]{8}=2$ feet, *Ans.*

---

examine our cube, with all these additions made and placed to it, we shall discover in one corner a vacancy, the length, breadth and thickness, of which is just 3 inches; (that is, the same as the thickness of our last addition; which, when filled, will just complete the cube. This vacancy contains $3\times3\times3=27$ cubic inches; that is, the cube of the last quotient figure. These 27 cubic inches we place under the former products, then add them up, and subtract the amount from the dividend, 4167, and 0 remains. Hence we find, that the side of a cube which contains 12167 inches must measure 23 inches; or, that the cube root of 12167 is 23.

Proof.—23^3, that is, $23\times23\times23=12167$, the given sum, therefore right.

Note. The solid contents of similar figures are in proportion to each other as the cubes of their similar sides or diameters.

6. If a ball 4 inches in diameter weigh 12 pounds, what will another ball of the same metal weigh, whose diameter is 7 inches?

$$\left.\begin{array}{l}4\times4\times4=\ 64\\7\times7\times7=343\end{array}\right\}\text{Then,}\quad \begin{array}{ccccccc}\text{in.} & & \text{lbs.} & & \text{in.} & & \text{lbs.}\\ 64 & : & 12 & :: & 343 & : & \textit{Ans. } 64\tfrac{5}{16}.\end{array}$$

7. If a globe of silver 3 inches in diameter be worth 150 dollars what is the value of a globe 8 inches in diameter? *Ans.* \$2844,44+

8. How many globes 1 foot in diameter would it take to make a globe 2 feet in diameter? *Ans.* 8.

9. The diameter of a ball weighing 4 pounds, is three inches; what is the diameter of another ball 8 times as large?

$3\times3\times3=27$, and $27\times8=\sqrt[3]{216}=6$ inches, *Ans.*

10. If the side of a cube of silver worth 20 dollars, be 2 inches, what is the side of another cube of silver, whose value shall be 64 times as much? *Ans.* 8 inches.

11. If the diameter of the earth is 8000 miles, and the sun is one million times as large as the earth, what is the diameter of the sun? *Ans.* Eight hundred thousand miles.

PROB. 1.—The product of two or more parts of any number given, to find that number.

RULE.

Divide the given product by the product of the given parts, and the quotient will be that power of the required number which is equal to the number of parts.

Ex. 1. If $\frac{1}{2}$ and $\frac{3}{4}$ of a certain number be multiplied together the product will be 54; what is that number?

Thus, $\frac{1}{2}\times\frac{3}{4}=\frac{3}{8}$, then $54\div\frac{3}{8}=144$, which is the 2d power of the required number, because the number of parts multiplied were 2; then $\sqrt{144}=12$, *Ans.*

Ex. 2. If $\frac{2}{3}$, $\frac{4}{5}$ and $\frac{5}{6}$ of a certain number be multiplied together the product will be 12000: what is that number?

Thus, $\frac{2}{3}\times\frac{4}{5}\times\frac{5}{6}=\frac{40}{90}=\frac{4}{9}$; then $12000\div\frac{4}{9}=27000$, which is the 3d power of the required number; and $\sqrt[3]{27000}=30$, the *Answer.*

PROB. 2. The product of any two or more numbers, and the proportion between them given, to find those numbers.

RULE.

Divide the given product by the product of the given terms of the proportion, and the quotient will be a power equal to the number of terms multiplied for a divisor; and the root of that power, multiplied severally by the given terms of the proportion, will produce the required numbers.

Ex. 1. The product of two numbers is 2240, and they are in proportion to each other as 5 to 7; what are these numbers? $5\times7=35)2240(64$, and $\sqrt{64}=8$;

Then 8×5, (one of the terms of the proportion,) gives - - - - - 40 } *Ans.*
And 8×7, (the other term of the proportion,) gives - - - - - - 56 } *Ans.*

Ex. 2. The product of three numbers is 1296, and they are in proportion to each other as 1, 2 and 3; what are those numbers?

Thus, $1\times2\times3=6)1296(216$,

then $\sqrt[3]{216}=6$ - - $6\times\begin{cases}1=6\\2=12\\3=18\end{cases}$ *Ans*

A GENERAL RULE FOR EXTRACTING ROOTS OF ALL POWERS.

RULE.

1. Prepare the given number for extraction, by pointing it off from the units' place, as the required root directs.
2. Find the first figure of the root by trial, and subtract its power from the left hand period.
3. Bring down the first figure in the next period to the remainder and call this the dividend.
4. Involve the root to the next inferior power to that which is given, and multiply it by the number denoting the given power for a divisor.
5. Find how many times the divisor may be had in the dividend, and the quotient will be another figure of the root.
6. Involve the whole root to the given power, and subtract that power from as many periods of the given number, as you have found figures in the root.

7. Bring down the first figure of the next period to the remainder for a new dividend.

8. Involve the whole root to the next inferior power to that which is given, and multiply it by the number denoting the given power for a divisor, as before; and proceed in this manner till the whole is finished.

Note. When the number to be subtracted is greater than the periods from which it is to be subtracted, the last quotient figure must be taken less, &c.

EXAMPLES.

1. What is the cube root of 94818,816?

$$
\begin{array}{rl}
 & \dot{9}48\dot{1}8,81\dot{6}(45,6 \text{ root.} \\
 & 64 \\
4\times4\times3=48) & \overline{308} \text{ dividend.} \\
45\times45\times45= & \overline{91125} \text{ subtrahend.} \\
45\times45\times3=6075) & \overline{3693,8} \text{ dividend.} \\
456\times456\times456= & \overline{94818,816} \text{ subtrahend.} \\
 & \overline{00000,000}
\end{array}
$$

2. What is the sursolid, or 5th root of 17210368?

$$
\begin{array}{rl}
 & 1\dot{7}21036\dot{8}(28 \text{ root.} \\
 & 32 \\
2\times2\times2\times2=16\times4=64) & \overline{1401} \text{ dividend.} \\
28\times28\times28\times28\times28= & \overline{17210368} \text{ subtrahend.} \\
 & \overline{00000000}
\end{array}
$$

Note. The roots of most powers may be found by the square and cube roots only, by the following

RULE.

1. For the biquadrate, or 4th root, extract the square root of the square root.

2. For the 6th root, extract the cube root of the square root.

3. For the 8th root, extract the square root, which reduces it to the 4th power; then extract the root of that power as above.

4. For the 9th root, extract the cube root of the cube root.

5. For the 12th root, extract the square root, which will reduce it to the 6th power; then find the root of the 6th power as above.

EXAMPLES.

1. What is the biquadrate, or 4th root of 20736

Thus, the square root of 20736 is 144;

then, the square root of 144 is 12, the *Ans*.

2. What is the square cubed, or the sixth root of 481890304?

Thus, $\sqrt{481890304}=21952$; and $\sqrt[3]{21952}=28$, *Ans*.

3. Extract the square biquadrate, or eighth root of 1001129150390625.

Thus, $\sqrt{1001129150390625}=\overset{\text{4th power.}}{\sqrt{31640625}}=\overset{\text{2d power, or square}}{\sqrt{5625}}=$ 75, root, the answer.

Questions.

1. What is a cube?
2. What is it to extract the cube root?
3. What is a solid body, having six equal sides, each an exact square?
4. What is the Rule for extracting the cube root?
5. When there is a remainder after bringing down all the periods, how may we continue the operation?
6. Repeat the rule for extracting the roots of all powers?
7. How may the roots of most powers be found?

ALLIGATION.

Alligation is the method of mixing several simples of different qualities, so that the composition may be a mean, or middle quality. It consists of two kinds, Alligation Medial and Alligation Alternate.

ALLIGATION MEDIAL,

Is when the quantities and prices of several things are given, to find the mean price of the mixture composed of those materials.

RULE.

As the sum of the quantities, or whole composition, is to the whole value, so is any part of the composition to its mean price, or value.

EXAMPLES.

1. A farmer mixed together 4 bushels, of rye, worth 90 cents per bushel, 6 bushels of corn, worth 50 cents per bushel, and 8 bushels of oats, worth 30 cents per bushel; what is a bushel of this mixture worth?

		cts.		cts.
4	bushels at	90	cost	360
6	"	50	"	300
8	"	30	"	240
18 bushels cost			- -	900

As 18 : 900 : : 1 : 50 cents, price of 1 bushel.

2. A grocer mixed 5cwt. of sugar, worth $10 per cwt., 8cwt. worth $12 per cwt., with 3cwt, worth $9 per cwt.; what will be the cost of 4cwt. of this mixture?

Ans. $43,25.

3. A vintner mixed together 16 gallons of wine at $1,12c. per gallon, 12 gallons at $,90 per gallon, and 20 gallons at $,95 per gallon; what is the price of a gallon of the mixture? *Ans.* $,99, 4m.+

4. A goldsmith melted together 5 ounces of gold 21 carats fine, and 3 ounces 19 carats fine; what is the fineness of the mixture, that is, of one ounce of this mixture?

Ans. 20¼ carats fine.

5. A Grocer mixed 20 gallons of water with 85 gallons of rum, worth 76 cents per gallon, what is a gallon of the mixture worth? *Ans.* 61cts. 5m.+

6. A Refiner melted together 12lb. of silver bullion of 6oz. fine, 8lb. of 7oz. fine, and 10lb of 8oz. fine, I demand the fineness of the mixture. *Ans.* 6oz. 18pwt. 16gr.

7. Suppose that 3lbs. of gold of 22 carats fine, 5lbs. of 20 carats fine, and 1lb. alloy be melted together, what will be the fineness of the compound? *Ans.* 18⅔ carats fine.

ALLIGATION ALTERNATE,

Is when the prices of the several simples and the mean price or rate are given, to find what proportion of each must be taken to compose a mixture of the given rate. It is therefore the reverse of Alligation medial, and may be proved by it.

CASE I.

When the mean rate and the rates of the several ingredients are given, without any limited quantity.

RULE.

1. Reduce the several prices to the same denomination.

2. Set the several prices under each other in a column, and place the mean rate on the left.

3. Connect or link each price which is less than the mean rate, with one or any number of those which are greater than the mean rate, and each price greater than the mean rate with one, or any number of the less.

4. Place the difference between the mean rate and that of each of the simples, opposite the price with which they are connected.

5. Then if only one difference stand against any rate, it will be the quantity belonging to that price, but if there be several, their sum will be the quantity.

EXAMPLES.

1. A Grocer would mix the following qualities of sugar, viz: at 8cts., 9cts., 11cts., and 12cts. per lb., what quantity must he take of each sort, that the mixture may be worth 10cts. per lb.

$$\begin{array}{ll} & \begin{array}{cc}\text{cts.} & \text{lbs.}\end{array} \\ \begin{array}{r}\text{cts.}\\ 10\end{array} & \left\{\begin{array}{cc} 8 & 2 \\ 9 & 1 \\ 11 & 1 \\ 12 & 2 \end{array}\right\}\textit{Ans.} \end{array} \qquad \text{Or} \qquad \begin{array}{l} \begin{array}{cc}\text{cts.} & \text{lbs.}\end{array} \\ \left\{\begin{array}{cc} 8 & 1 \\ 9 & 2 \\ 11 & 2 \\ 12 & 1 \end{array}\right\}\textit{Ans.} \end{array}$$

Here we set down the prices of the simples in order, from the least to the greatest; placing the mean rate at the left hand. And in the first way of linkng the prices, and taking the difference between the several prices and mean rate, and placing each difference at the opposite end of the link, we find that we must take in proportion of 2lbs. at 8cts., 1lb. at 9cts., 1lb. at 11cts., and 2lbs. at 12cts.; and in the 2d operation, we have the proportion of 1lb. at 8cts., 2lb. at 9cts., 2lb. at 11cts., and 1lb. at 12cts. It will be seen that by linking any two of the prices together, and placing the

difference between those prices and the mean rate alternately, that is, placing the difference between the greater price and the mean rate, against the lesser price, and placing the difference between the lesser price and the mean rate, against the greater price, the difference of the prices become mutually changed; and these differences express the relative quantities of each simple, necessary to form the compound, and what is lost on one quantity, is gained on another. Hence the balance of loss and gain between any two, will be equal, consequently the same on the whole.

2. A Merchant has three sorts of tea, viz.: one sort at 5 shillings per lb., another at 7 shillings per lb., and another at 8 shillings per lb., what proportion of each kind must he take to make a mixture worth 6s. per lb.?

$$\text{Ans.}\ \overset{\text{s.}}{6}\left\{\begin{array}{l} \overset{\text{s.}}{5} \\ 7 \\ 8 \end{array}\right. \begin{array}{l} 2+1 = 3\text{lb. at 5s.} \\ 1 \quad\ = 1\text{lb. at 7s.} \\ 1 \quad\ = 1\text{lb. at 8s.} \end{array}$$

3. How much vinegar at 42cts., 60cts. and 67cts. per gallon, must be mixed together, so that it may be worth 64 cents per gallon?

Ans. 3gals. at 42cts., 3do. at 60c. and 26do. at 67c.

4. How much sugar at 9cts. and 15cts. per lb. must be mixed together, so that the compound may be worth 12 cents per lb.? *Ans.* An equal quantity of each sort.

5. A Goldsmith mixed together gold of 17, 19, 21, and 24 carats fine, so that the composition was 22 carats fine what proportion did he take of each?

Ans. 2 of each of the first 3 sorts, and 9 of the last.

6. A Merchant has spices at 7d. 8d. 10d. and 11d. per lb. which he would mix together so that the whole composition may be sold at 9d. per lb., what proportion must he take of each kind?

1st Ans.			2d Ans.			3d Ans.		
	2lb.	at 7cts.		1lb.	at 7cts.		3lb.	at 7cts.
	1	8		2	8		2	8
	1	10		2	10		2	10
	2	11		1	11		3	11

4th Ans. 3lb. of each sort.

These four answers arise from the various ways of link

ing the prices of the ingredients together. Hence we may see that questions in this rule admit of a great variety of answers ; for having found one answer, we may take any other numbers which have the same proportion between themselves, as the numbers have which compose the answer.

CASE II.

When one of the ingredients is limited to a certain quantity to find what quantity of each of the others must be taken in proportion to the given quantity.

RULE.

1. Link the prices and take their differences as in Case I. which will give the unlimited proportions.

2. Then as the proportion whose quantity is limited, is to its limited quantity, so is each of the other proportions to its required quantity.

EXAMPLES.

1. A man wishes to mix 5 bushels of wheat worth 90 cents per bushel, with rye at 56cts. per bushel, barley at 36cts., and oats at 30cts., so that the composition may be sold for 45cts. per bushel.

	90	15	the proportion whose quantity is limited.
cts.	56	9	
mean rate 45	36	11	
	30	45	

then as 15 : 5 :: 9 : 3 bushels of rye
15 : 5 :: 11 : $3\frac{2}{3}$ " barley.
15 : 5 :: 45 : 15 " oats.

2. How much water at 0 per gallon, must be mixed with 100 gallons of brandy worth 180cts. per gallon, to reduce it to 150cts. per gallon? *Ans.* 20gal.

3. How much gold of 15, of 17, and of 22 carats fine, must be mixed with 5oz. of 18 carats fine, so that the composition may be 20 carats fine?

Ans. 5oz. of 15, 5oz. of 17, and 25oz. of 22 carats fine.

4. With 95 gallons of wine worth 96cts. per gallon, I mixed wine at 80cts. per gallon, and some water, so that it

stood me in 76cts per gallon, how much wine and how much water did I take?

Ans. 95gals. wine at 80cts., and 30gals. water

CASE III.

When the whole composition is limited to a given quantity.

RULE.

1. Link the prices and find the proportions as in Case 1. Then, as the sum of the proportion is to the given quantity or whole composition, so is each proportion to its required quantity.

EXAMPLES.

1. A Grocer has sugar at 5cts., 7cts., 10cts. and 13cts. per lb., and would mix them together so as to fill a cask of 200lbs. worth 8cts. per pound.

	cts.	
	5	5
cts.	7	2
mean rate, 8	10	1
	13	3
		11 = sum of the proportion.

As 11 : 200 : : 5	$90\frac{10}{11}$lbs.	at 5cts.
11 : 200 : : 2	$36\frac{4}{11}$	7cts.
11 : 200 : : 1	$18\frac{2}{11}$	10cts.
11 : 200 : : 3	$54\frac{6}{11}$	13cts.

2. How much water of no value must be mixed with spirits at 90cts. per gallon, so as to fill a cask of 120gals. that may be sold for 60cts. per gallon?

Ans. 40gals. of water, and 80gals. of spirits.

3. A Grocer has teas at 2s., 3s., 5s., and 6s. per lb. and would mix them together so that the composition may be worth 4s. per lb. What quantity must he take of each, to fill a chest that will hold 90lbs.?

Ans. 30lbs., 15lbs., 15lbs., and 30lbs

4. How much gold of 15, of 17, of 18, and of 22 carats fine, must be mixed together, to make a composition of 40oz. that will be 20 carats fine?

Ans. 5oz. of 15 5 of 17, 5 of 18, and 25oz. of 22

Questions.

1. What is Alligation?
2. What is Alligation Medial?
3. What is the rule?
4. What is Alligation Alternate?
5. When the mean rate and the rates of the ingredients are given without any limited quantity, how do you find the proportional quantity?
6. When one of the ingredients is limited to a certain quantity, how do you find what quantity of each of the others must be taken in proportion to the given quantity?
7. When the whole composition is limited to a given quantity, how do you find the proportional quantities of each ingredient?

ARITHMETICAL PROGRESSION.

Any rank, or series of numbers more than two, increasing by a common excess, or decreasing by a common difference, is called an Arithmetical Series or Progression.

The number which is continually added or subtracted, is called the common difference.

When the numbers are formed by a continual addition of the common difference it is called an ascending series; but when they are formed by a continual subtraction of the common difference, they form a descending series.

Thus, $\begin{cases} 2,\ 4,\ 6,\ 8,\ 10, \text{ is an ascending series.} \\ 10,\ 8,\ 6,\ 4,\ 2, \text{ is a descending series.} \end{cases}$

The numbers which form the series are called the terms of the series, or progression; the first and last terms of which are called the extremes.

A series in progression includes five parts, viz.:

1. The first term.
2. The last term.
3. The number of terms.
4. The common difference.
5. The sum of all the terms:

by having any three of which given, the other two may be found.

CASE I.

The first term, common difference, and number of terms given, to find the last term.

RULE.

Multiply the number of terms, less 1, by the common difference, and to the product add the first term, and the sum will be the last term.

EXAMPLES.

1. A man bought 17 yards of cloth, giving 5 cents for the first yard, 8 cents for the second, 11 cents for the third, and so on, increasing with a common difference of three cents on each yard; what was the cost of the last yard?

Numb. of terms, less 1,	$=16$
	$\times 3$
	48
First term, - - -	$+5$
Last term, - - -	$=53$

It will be seen that the common difference will always be added one time less than the number of terms. Thus, for the second yard the common difference is added to the first yard one time, for the third yard it is added 2 times; and for the 17th yard, it must be added to the price of the first yard 16 times. $16\times 3=48$, that is the 17th yard will cost 48 cents more than the 1st yard; and $48+5=53$, the price of the last.

2. If the first term be 5, and the common difference 3, and the number of terms 100, what is the last term?

Ans. 302.

3. A man, in traveling a certain journey in 11 days, traveled thirteen miles the first day, and increased every day two miles; how many miles did he travel the last day?

Ans. 33 miles.

CASE II.

The first term, last term, and number of terms given, to find the common difference.

RULE.

Divide the difference of the extremes by the number of terms less 1, and the quotient will be the common difference.

EXAMPLES.

1. A man bought 17 yards of cloth in arithmetical progression. For the first yard he gave 5 cents, and for the last 53 cents; what was the common difference, or the increase of the price on each succeeding yard?

Extremes $\begin{cases} 53 \\ -5 \end{cases}$

—

No. of terms, less 1, } 16)48(3 com. diff.

This operation is exactly the reverse of those in Case I, and taking the 1st extreme from the last evidently shows the whole increase of the 16 additions; and dividing by 16, the number of terms, less 1, shows the increase of one addition; that is, the common difference.

2. If the extremes be 3 and 273, and the number of terms 46, what is the common difference? *Ans.* 6.

3. A man had 8 sons whose several ages differed alike; the youngest was 7 years old and the oldest 35; what was the common difference of their ages? *Ans.* 4yrs.

4. A man performed a journey in 11 days. The first day he traveled thirteen miles and the last day 33, increasing every day by an equal excess; what was the daily increase? *Ans.* Daily increase 2 miles.

CASE III.

The first term, last term, and number of terms given, to find the sum of all the terms.

RULE.

Multiply the sum of the extremes by the number of terms, and half the product will be the answer.

EXAMPLES.

1. A man bought 12 yards of cloth in arithmetical progression. For the first yard he gave 6 cents, and for the last yard 39 cents; what did the whole amount to?

Operation.

	6
	39
Sum of the extremes	45
Number of terms	×12
	)540
	270c.

If we take half the price of the first and last yards it will give the average price of the whole. Thus, $6+39=45$, the sum of the extremes, and $45\div2=22\frac{1}{2}$cts. the average price, and $22\frac{1}{2}\times12=270$cts., the *Ans.*

But if we multiply the whole sum of the extremes by the number of terms, (as in the operation,) and take half

the product, it will produce the same effect, and will generally be found preferable, as it will prevent the necessity of fractions. Hence the Rule.

2. If the first term of an arithmetical series be 3, the last term 150, and the number of terms 50, what is the sum of the series? *Ans*. 3825.

3. How many strokes does a regular clock strike in 12 hours? *Ans*. 78.

4. What is the sum of the first 25 numbers in their natural order? thus, 1, 2, 3, 4, 5, &c. *Ans*. 325.

5. A merchant sold 100 yards of cloth. For the first yard he received 12 cents, for the second 15cts., for the third 18 cents, and so on, increasing 3 cents on each succeeding yard to the last; what did he receive for the last yard, and what was the amount of the whole?

(*Note*. We must find the last term by the Rule, Case I. Thus, $3\times99+12=309$.)

Ans. Last yard \$3,09; whole amount \$160,50.

6. A man performed a journey in 11 days, traveling 13 miles the 1st day, and increasing every day by an equal excess until the last day's travel was 33 miles: what was the daily increase, and the whole distance traveled?

(The daily increase or common difference is found by Case 2.) *Ans*. Daily increase 2m., whole distance 253m.

7. If the first term in an arithmetical progressien be 4, the second 7, the third 10, &c. up to the 81st term, what is the sum of all the terms? *Ans*. 10044

8. If 140 oranges be placed in a straight line 2 yards distant from each other, and a basket placed 2 yards distant from the first orange, what distance must that boy travel who gathers them up singly, returning with them 1 by 1 to the basket?

(*Obs*.—It will be seen that the travel to the 1st orange and back again to the basket will be $2+2=4$ yards, and to the last orange and back again to the basket will be $4\times140=560$ yards. Hence the first term is 4, the last term 560, and the number of terms 140.)

Ans. 22 miles 3 furlongs 100 yards.

CASE IV.

The first term, last term and common difference given, to find the number of terms.

RULE.

Divide the difference of the extremes by the common difference, and the quotient increased by 1, will be the number of terms.

EXAMPLES.

1. A man bought cloth in arithmetical progression, giving 5 cents for the first yard, and increasing the price of each succeeding yard by 3 cents to the last, which was 53 cents; how many yards did he buy?

Extremes	53
	−5
Common difference	3)48
Number of terms, less 1,	16
	+1
Ans.	yards 17

We found (Case 2, Ex. 1, that the difference of the extremes, divided by the number of terms, less 1,) gave the common difference, and this question being the reverse, it is evident that the difference of the extremes, divided by the common difference, will give the number of terms, less 1.

2. If the extremes be 3 and 273, and the common difference 6, what is the number of terms? *Ans.* 46.

3. A man being asked how many children he had, replied, that his youngest was 7 years old and his oldest 35, and the common difference of their ages was four years; how many had he? *Ans.* 8

4. A man going a journey traveled the first day thirteen miles, and increased every day's travel two miles to the last, which was 33 miles; how many days did he travel, and how far did he travel?

Ans. 11 days, and the distance traveled 253 miles.

Questions

1. What is Arithmetical Progression?
2. What is the number which is continually added or subtracted called?
3. When the numbers are formed by a continual addition of the common difference, what is it called?
4. When formed by a continual subtraction, what is it called?
5. What are the numbers called which form the series?
6. What are the first and last terms called?
7. How many parts does a series in progression include, and what are they?
8. Having the first term, common dif

ference and number of terms given, how do you find the last term ?

9. Having the first term, last term, and the number of terms given, how do you find the common difference ?

10. Having the first term, last term, and number of terms given, how do you find the sum of all the terms ?

11. Having the first term, last term and common difference, how do you find the number of terms ?

GEOMETRICAL PROGRESSION,

Is any rank or series of numbers, increasing by a constant multiplier, or decreasing by a constant divisor; and this multiplier or divisor is called the ratio of the progression.

Thus, { 1, 2, 4, 8, 16, &c., is an increasing geometrical series; and 8, 4, 2, 1, ½ &c., is a decreasing geometrical series, and the ratio is 2. }

In this, as in arithmetical progression, there are five parts, viz.: 1st, the first term; 2nd, the last term; 3d, the number of terms; 4th, the ratio; 5th, the sum of all the terms, any 3 of which being given, the other two may be found.

CASE I.

Given, the first term, the ratio, and the number of terms, to find the last term.

RULE.

1. Write down a few of the leading powers of the ratio, placing their indices,* viz.: 1, 2, 3, 4, &c. over them.

2. Add together the most convenient indices, to make an index, less by 1 than the number of the term sought.†

3. Multiply together the powers belonging to, or standing under, those indices, and their product, multiplied by the first term, will give the term sought.

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* To find the last term of a long series of numbers, by multiplication, would be very tedious, therefore we have a series of numbers in arithmetical proportion, called indices, whose common difference is 1. Then if the powers standing under any of those indices be multiplied together, their product will express a power equal to the sum of those indices which were used.

† The reason why the sum of the indices must be 1 less than the number of terms sought is very evident: for example, if the first term be 4, and the ratio 3, required the sixth term. Thus, $4\times3\times3\times3\times3\times3=972$, the sixth term; and if we examine the process, we shall see that the ratio is five times a factor; that is, one time less than the number of terms; and the fifth power of the ratio, mul

EXAMPLES.

1. A man bought 12 yards of cloth, and by agreement was to pay what the last yard would come to, at 3 cents for the 1st yard, 6 cents for the second, and so on, doubling the price to the last; what did the last yard come to?

1	2	3	4	5	indices.
2	4	8	16	32	leading powers of the ratio.

Then, $5+4+2=11$, the number of terms, less 1.

$32 \times 16 \times 4 = 2048$, the 11th power of the ratio.

$\times 3$, the first term.

The last term, or cost of the last yard, } 6144cts.$=$\$61,44, *Ans.*

2. If the first term be 4, and the ratio 3, what is the 25th term? *Ans.* 1129718145924.

3. A drover purchased 15 head of cattle, and agreed to pay what the last would come to, reckoning 3 dollars for the first, 12 dollars for the second, and so on in a quadruple, or fourfould proportion; I demand the sum to be paid? *Ans.* \$805306368.

4. A draper sold 20 yards of superfine cloth, on condition that he should receive pay for the last yard only, reckoning 3 cents for the first yard, 9 cents for the second, 27 cents for the third, and so on in triple proportion; how much did he receive for the whole? *Ans.* \$34867844,01.

5. Twelve men received a sum of money. The first had \$3, the second \$6, the third \$12, and so on in a twofold proportion; how many dollars had the last? *Ans.* \$6144.

6. If the first term of a geometrical series be 1, and the ratio 6, what is the 12th term? *Ans.* 362797056.

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tiplied by the first term, produces the sixth. Hence the sum of the indices used denoting the leading powers of the ratio, must always be 1 less than the number of the terms sought.

Again; if the first term be 3, and the ratio 2, what is the 13th term?—Thus, $3\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2\times2=12288$. Here we see that the ratio is twelve times a factor, and multiplied into the first term; that is, the 12th power of the ratio multiplied by the first term,$=$13th term.—But to raise the ratio to its 12th power, we need not multiply all the intermediate powers; for $2^4=16$ and $16\times16=256$, the 8th power; that is, the 4th power $\times$ 4th power$=$8th power, and $256\times16=4096$; the 12th power, and $4096\times3=12288$, as before.

CASE II.

The first term, the last term, (or the extremes,) and the ratio given, to find the sum of the series.

RULE.

Multiply the last term by the ratio; from the product subtract the first term, and divide the remainder by the ratio, less 1, and the quotient will be the sum of all the terms.

EXAMPLES.

1. A man bought 6 yards of cloth, giving 2 cents for the first yard, 6 cents for the second, and so on in triple proportion; what did the last yard cost, and how much was the cost of the whole?

By Case I, the cost of the last yard is 486 cents.

Then, $\frac{486\times3-2}{3-1}=728$ the *Ans.*

2. If the first term of a geometricial series be 3, and the last term 6144, and the ratio 2, what is the sum of all the terms? *Ans.* 12285.

3. If the extremes of a geometrical series be 10, and 196830, and the ratio 3, what is the sum of all the terms? *Ans.* 295240.

(*Note.* In the following examples, the scholar must find the last term of the series by Case I; then find the sum of all the terms, as above.)

4. A man purchased a valuable tract of land containing fifteen acres, agreeing to give 1 dollar for the first acre, 4 dollars for the second, and so on in a quadruple, or fourfold proportion; what did the whole tract cost him? *Ans.* $357913941

5. What debt can be discharged in a year, by paying 1 cent the first month, 10 cents the second, and so on in a tenfold proportion? *Ans.* $1111111111,11.

6. If a pound (12oz.) of gold be sold at the rate of 2 cents for the first ounce, 8 cents for the second, 32 cents for the third, and so on in a fourfold proportion to the last; what will it amount to? *Ans.* $111848,10.

7. A merchant sold 14 yards of Italian silk at the rate of 4 cents for the first yard, 12 cents for the second, and so

on in geometrical progression; how much did the last yard come to, and what did the whole amount to? *Ans.* The last come to $63772,92, and the whole $95659,36.

8. A man bought a horse, and by agreement was to give a cent for the first nail, two for the second, four for the third, and so on; there were four shoes, and eight nails in each shoe; what did the horse come to at that rate?

Ans. $42949672,95.

9. A thresher worked 20 days for a farmer, and received for the first day's work 4 barley-corns, for the second 12 barley-corns, for the third 36 barley-corns, and so on in triple geometrical progression; what did his 20 day's labour come to, allowing 7680 barley-corns to make a pint, and the whole quantity to be sold at 50cts. per bushel?

Ans. $7093,50, rejecting remainders.

10. If a body put in motion move 1 inch the first second of time, 3 inches the second, 9 inches the 3d second of time, and thus continue to increase its motion in triple proportion, geometrical, how many yards will it move in the term of half a minute? *Ans.* 2859599056870yds. 0ft. 4 inches, which is no less than one thousand six hundred and twenty-four millions of miles.

Questions.

1. What is Geometrical Progression?
2. What is the multiplier or divisor in Geometrical Progression called?
3. How many parts are there in Geometrical Progression?
4. Having the first term, the ratio, and the number of terms given, how do you find the last term?
5. Having the first term, the last term, (or the extremes,) and the ratio given, how do you find the sum of the series?

ANNUITIES.

To find the amount of an annuity at Simple Interest, by Arithmetical Progression.

RULE.

Make 1 the first term of an arithmetical series, and the ratio the common difference—multiply the common difference by the number of terms less 1, and to the product add the first term for the last. Then find the sum of all the terms by the Rule, Case III. page 205, which will be the amount of $1 annuity for the given number of years Multiply this amount by the given sum for the whole amount

EXAMPLES.

1. What will \$200 yearly annuity, remaining unpaid or in arrears 8 years, amount to at 6 per cent?

	Ratio ,06	then first term	1,00
Multiply by 8 years less 1=7		last term	1,42
	,42	sum	2,42
Add the first term,	1,00	number of terms	8
	1,42		2)19,36
Amount of \$1 annuity for 8 years,			9,68
			200
		Ans.	\$1936,00.

2. What is the amount of a yearly rent of \$75 remaining unpaid, or in arrears, 25 years, at 6 per cent? [*Ans.* ,06×24+1=2,44. Then 1+2,44×25÷2×75=\$3225,

3. What is the amount of an annuity of \$600 remaining in arrears 10 years, at 6 per cent? *Ans.* \$7620.

ANNUITIES OR PENSIONS COMPUTED AT COMPOUND INTEREST.

CASE I.

To find the amount of an annuity or pension in arrears at Compound Interest.

RULE.

Raise the ratio to a power equal to the given number of years. From that power subtract 1, and divide the remainder by the ratio, less 1, and the quotient will be the amount of \$1 annuity for the given time. Multiply this amount by the given annuity, and the product will be the amount required.

Note. The ratio is the amount of \$1, &c. at the given rate for one year.

EXAMPLES.

1. What will \$60 yearly annuity amount to, being forborne, or unpaid 5 years, at 6 per cent compound interest.

Ratio 1,06×1,06×1,06×1,06×106—1,÷,06=5,63709+.

Amount of \$1 annuity for 5 years=	5,63709
given annuity	×60
Amount of \$60 annuity 5 years	\$338,22540

A TABLE.

Showing the amount of $1, or £1 annuity at 5 and 6 per cent for any number of years, not exceeding 34.

yrs.	5 per cent.	6 per cent.	yrs.	5 per cent.	6 per cent.
1	1,000000	1,000000	18	28,132385	30,905653
2	2,050000	2,060000	19	30,539004	33,759992
3	3,152500	3,138600	20	33,065954	36,785592
4	4,310125	4,374616	21	35,719252	39,992727
5	5,525631	5,637093	22	38,505214	43,392291
6	6,801913	6,975318	23	41,430475	46,995828
7	8,142009	8,393837	24	44,501999	50,815578
8	9,549109	9,897467	25	47,727099	54,864512
9	11,026564	11,491315	26	51,113454	59,156382
10	12,577892	13,180770	27	54,669126	63,705765
11	14,206787	14,971643	28	58,402583	68,528112
12	15,917126	16,869942	29	62,322712	73,639798
13	17,712983	18,882138	30	66,438847	79,058186
14	19,598632	21,015066	31	70,760790	84,801677
15	21,578564	23,275969	32	75,298829	90,889778
16	23,657492	25,672528	33	80,063771	97,343165
17	25,840366	28,212380	34	85,066959	104,183754

To find the amount of an annuity by this Table.

RULE.—Opposite the given year and under the rate per cent you will find the amount of $1 for the given time. Multiply this amount by the given annuity.

2. If a salary of $90 per annum, to be paid yearly, remain in arrears 12 years, what will be its amount at 5 per cent compound interest?

Tabular amount 15,917126
given annuity ×90

Ans. $1432,541340=$1432,54c. $1\frac{34}{100}$m.

3. What is the amount of an annuity of $100 remaining unpaid, or in arrears 20 years, at 6 per cent compound interest? *Ans.* $3678,55c.$9\frac{2}{10}$m.

4. If an annual pension of $130 remain in arrears 7 years, what is the amount due, at 6 per cent compound interest? *Ans.* $1091,19c.$8\frac{81}{100}$m.

5. What would $50 annuity amount to at 6 per cent compound interest, for 33 years? *Ans.* $4867,15c.8+m.

CASE II.

To find the present worth of annuities.

The present worth here spoken of, is such a sum as if put at compound interest for the given rate and time will be equal to the amount of the given annuity.

RULE.

Raise the ratio to a power equal to the given number of years. Then divide the annuity by that power, and subtract the quotient from the annuity, and divide the remainder by the ratio, less 1.

EXAMPLES.

1. What sum of ready money will purchase a yearly annuity of $60, to continue 4 years, at 6 per cent?

Ratio 1,06×1,06×1,06×1,06=
1,26247696)60,00000000000(47,5256+
then from the annuity 60,0000
subtract this quotient 47,5256
ratio 1,06—1=,06)12,4744
Ans. $207,906+=$207,90c. 6m.+

TABLE II.

Showing the present worth of an annuity of $1 *or* £1, *at* 5 *and* 6 *per cent for any number of years from* 1 *to* 32.

yrs	5 per cent.	6 per cent.	yrs.	5 per cent.	6 per cent.
1	0,952381	0,943396	17	11,274066	10,477260
2	1,859410	1,833393	18	11,689587	10,827603
3	2,723248	2,673012	19	12,085321	11,158116
4	3,545950	3,465106	20	12,462210	11,469921
5	4,329477	4,212364	21	12,821153	11,764077
6	5,075692	4,917324	22	13,163003	12,041582
7	5,786278	5,582381	23	13,488574	12,303380
8	6,463213	6,209794	24	13,798642	12,550357
9	7,107822	6,801692	25	14,093944	12,783356
10	7,721735	7,360087	26	14,375185	13,003166
11	8,306414	7,886875	27	14,643034	13,210534
12	8,863252	8,383844	28	14,898127	13,406164
13	9,393573	8,852683	29	15,141073	13,590721
14	9,898641	9,294984	30	15,372451	13,764831
15	10,379658	9,712249	31	15,592810	13,929080
16	10,837769	10,105895	32	15,802681	14,084042

RULE.

Multiply the present worth, found in Table II. opposite he given year, and under the rate per cent, by the given annuity.

2. What is the present worth of an annuity of $200, to continue 10 years, at 6 per cent?

by Table II. present worth of $1 for 10yrs. 7,360087
×200

Ans. $1472,1c.7$\frac{4}{10}$m. $1472,017400

3. How much must be paid in ready money, to purchase an annuity of $156 to continue 8 years at 5 per cent?

Ans. 1008,26c.1m.+

4. What is the present worth of an annual pension of $96 a year, to continue 5 years at 6 per cent compound interest? *Ans.* $404,38c.6m.+.

5. What ready money will purchase an annuity of $112 to continue 30 years, at 5 per cent compound interest?

Ans. $1721,71,4$\frac{5}{10}$+.

CASE III.

To find the present worth of annuities, leases, &c. taken in reversion at Compound Interest.

Observation.—Annuities in reversion, are those which do not commence till some particular event has happened, or until the expiration of a certain time.

RULE.

1. Divide the annuity by the power of the ratio equal to the time of its continuance, and subtract this quotient from the annuity, and divide the remainder by the ratio less 1, and the quotient will be the present worth to commence immediately.

2. Divide the quotient by the power of the ratio equal to the time of reversion, (or the time to come before the annuity commences,) and the quotient will be the present worth of the annuity in reversion.

1. What is the present worth of $200, payable yearly for 4 years but not to commence, (that is being in reversion,) till the end of 2 years, at 6 per cent?

4th power of 1,06=1 26247696)200,00000000(158,418732
158,418732

Ratio 1,06 less 1, ,06)41,581268
$ cts.m.

2d power of 1,06 1,1236)693,0211(616,78,6+

It will be much easier to find the present worth of annuities in reversion, by 2d Table. Thus, find the present worth of $1, or £1, annuity for the sum of the time of reversion and the time of continuance added together; and from this present worth, subtract the present worth of $1 or £1 for the time of reversion, and multiply the remainder by the given annuity.

2. What is the present worth of $50 yearly annuity to continue 8 years, and to be in reversion 3 years at 5 per cent? Time of reversion, or the time before the annuity commences 3 years.
Time of continuance 8 years.

the sum 11

The present worth of $1 for 11 yrs. by 2d Table, 8,306414
" " 3 yrs. reversion 2,723248

Remainder 5,583166
×50

Ans. $279,158300

3. What is the present worth of the reversion of a lease of $125 to continue 20 years, but not to commence till the end of 9 years, allowing 6 per cent?

Ans. $848,62c.8+m.

4. What sum of ready money will purchase the reversion of an annuity of $60 to continue 15 years, but not to commence till the end of 10 years, at 5 per cent?

Ans. $382,33c.2+m.

CASE IV.

To find the present worth of freehold estates, or annuities to continue forever.

RULE.

As the rate per cent is to 100, so is the yearly rent or annuity to the value required. Or, divide the yearly rent

or annuity by the rate per cent; and the quotient will be the annuity.

EXAMPLES.

1. What is the worth of a freehold estate of which the yearly rent is $50, allowing to the purchaser 5 per cent?

5 : 100 : : 50 $1000 *Ans.*

or, ,05)50,00(1000 Answer as before.

2. What is the worth of $125 annuity to continue forever, allowing 6 per cent to the purchaser?

Ans. $2083,33⅓.

3. What is the value of an estate which brings in yearly $96, allowing 6 per cent to the purchaser? *Ans.* $1600.

CASE V.

To find the present worth of perpetual annuities or freehold estates in reversion at Compound Interest.

RULE.

Find the present worth of the estate by the Rule, Case 4th, which will give the value if entered on immediately. Then divide that value by the power of the ratio denoted by the time of reversion, and the quotient will be the present worth of the annuity in reversion.

EXAMPLES.

1. Suppose a freehold estate of $50 per annum to commence 2 years hence, be put on sale, what is its value, allowing the purchaser 5 per cent?

5 : 100 : : 50 $1000 present worth if entered on immediately.

the ratio1,05 2d power=1,1025)1000,0000(907,02,9+*Ans.* ($ $ cts.m.)

By Table 2, find the present worth of the annuity or rent for the time of reversion, which subtract from the value of the annuity found by Case IV.

By Tab. 2d, the present worth of $1 for 2yrs.		1,859410
		×50
Present worth for the time of reversion,		92,970500
Value of immediate possession,	1000,0000	
	92,9705	
Ans. the same as before,	$907,0295	

2. Suppose an estate of \$100 per annum to commence 6 years hence, were to be sold, allowing the purchaser 5 per cent what is its value? *Ans.* \$1492,43c.$0\frac{8}{10}$m.

3. What is the value of the reversion of an estate of \$120 per annum, to commence 15 years hence, at 6 per cent? *Ans.* \$834,53+

Questions.

1. How do you find the amount of annuity at simple interest, by Arithmetical Progression?
2. How do you find the amount of annuities or pensions in arrears, at compound interest?
3. How do you find the present worth of annuities at compound interest?
4. How do you find the present worth of annuities, leases, &c. taken in reversion, at compound interest?
5. How do you find the present worth of freehold estates or annuities forever?
6. How do you find the present worth of perpetual annuities, or freehold estates in reversion at compound interest?

PERMUTATION.

Permutation is a method of finding how many different ways any given number of things may be changed.

To find the number of different changes or permutations that can be made of any given number of things, different from each other.

RULE.

Multiply all the terms of the natural series continually together, from one up to the given number, and the last product will be the answer.

EXAMPLES.

1. How many changes can be made of the first three letters of the alphabet?

If there were but 2 letters, we could only change them $1\times2=2$ ways, thus, a, b, and b, a. But three letters can be changed $1\times2\times3=6$ different ways, as follows:

$$1\times2\times3=6 \textit{ Ans.} \begin{array}{r} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}\left\{\begin{array}{ccc} a & b & c \\ a & c & b \\ b & a & c \\ b & c & a \\ c & a & b \\ c & b & a \end{array}\right.$$

2. How many changes can be made with the nine digits? *Ans.* 362880.

3. Eight gentlemen agreed to dine together so long as they could sit every day in a different position; now admitting they had fulfilled their agreement, how long must they have tarried together? *Ans.* 40320 days, $=110\frac{34}{73}$yrs.

4. How many changes may be rung on 9 bells?—and how long will it take to ring them, allowing 20 seconds to every change? *Ans.* to the last, 84 days.

5. Of what number of variations will the twenty-six letters of the alphabet admit?

Ans. 403291461126605635584000000.

Questions.

1. What is Permutation?
2. How do you find the number of different changes or permutations that can be made of any given number of things?

POSITION

Is a Rule which, by the use of false, or supposed numbers, discovers the true ones required.

It is divided into two parts, Single and Double.

SINGLE POSITION.

Single Position teaches to solve those questions whose results are proportional to their suppositions.

RULE.

1. Take any number, and perform the same operations with it as are described to be performed in the question.
2. Then as the result of the operation is to the given sum, so is the supposed number to the true one required.

EXAMPLES.

1. A schoolmaster being asked how many scholars he had, replied: If I had as many more as I now have, half as many, one-third as many, and one-fourth as many, I should then have 222. How many scholars had he?

Suppose he had	12
Then as many more	12
$\frac{1}{2}$ as many	6
$\frac{1}{3}$ as many	4
$\frac{1}{4}$ as many	3
	37

Then, as 37 : 222 : : 12 :

$$\begin{array}{r} 12 \\ \hline 37)2664(72 \textit{ Ans.} \end{array}$$

Proof.

$$\begin{array}{r} 72 \\ 72 \\ 36 \\ 24 \\ 18 \\ \hline 222 \end{array}$$

In this example it is evident that, had we supposed the true number of scholars which he actually had, the amount would have been 222. But our supposed number has been increased in the same proportion as the true number, and consequently bears the same proportion to the true number as its result bears to the result of the true number; that is, 12 bears the same proportion to the true number, as 37 bears to 222, the true result.

2. A man having a certain number of dollars, said that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$ of them added together would make 114.—How many dollars had he?

Ans. \$120.

3. Divide 125 dollars among A, B, and C, so that B may have half as much as A, and C three times as mnch as B.

Ans. A's share \$$41\frac{4}{6}$; B's \$$20\frac{5}{6}$; and C's \$$62\frac{3}{6}$

4. A person having a certain sum of money, said that if $\frac{5}{6}$ of it be multiplied by 9, the product would be \$945; how many dollars had he? *Ans.* \$126.

5. A person lent his friend a certain sum of money, to receive interest for the same at 6 per cent per annum, simple interest, and at the end of twelve years, received, for principal and interest, \$774. What was the sum lent?

Ans. \$450.

6. A, B and C, gained by trading \$1360, of which A took a certain sum, B took $3\frac{1}{4}$ times as much as A, and C took as much as A and B both; how much did each have?

Ans. A had \$160, B \$520, and C \$680.

7. A man going to market with some sheep, cows and oxen, being asked how many he had of each, replied, that he had three times as many cows as oxen, and twice as many sheep as cows, and that one-fifth of his whole num

ber made 18; how many had he of each? *Ans.* 9 oxen, 27 cows, and 54 sheep.

DOUBLE POSITION

Teaches to solve questions by means of two suppositions, or false numbers.

In Single Position, the number sought is always multiplied, or divided, by some proposed number, or increased or diminished by itself, or a certain part of itself, and the result is always proportional to the supposition.

But in Double Position, the number sought is always increased by the addition, or decreased by the subtraction, of some number which is not a known proportional part of the true number. Hence the results are not proportional to the suppositions.

RULE.

1. Suppose any two convenient numbers, and proceed with each according to the conditions of the question, and find how much the results differ from the result in the question.

2. Multiply the first position by the last error, and the last position by the first error.

3. Then if the results be both greater or both smaller than the true number, divide the difference of these products by the difference of the errors, and the quotient will be the answer.

4. But if one result be greater, and the other less, than the true number, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

EXAMPLES.

1. A father gave his three sons 15000 dollars in the following manner; to the first he gave a certain sum, to the second he gave $3000 more than to the first, and to the third he gave as much as to the first and second both; how much did he give to each?

Sup. he gave the 1st	$1500
then the second	4500
and the third	6000
the result	= 12000
this subtracted from	15000
leaves the 1st error	3000
last position	×2000
	6000000
	1500000

Again, suppose the 1st	$2000
then the second	5000
and the third	7000
the result	= 14000
subtracted from	15000
leaves the 2d error	1000
first position	×1500
	1500000

Diff. of the prod. 4500000; this divided by 2000, the difference of the errors, gives $2250, the share of the first; to which + 3000=5250, the share of the second; and 2250+5250=7500, the share of the third.

2. B, C and D built a house which cost $1000. C paid $100 more than B, and D paid as much as B and C both; how much did each man pay?

Ans. B paid $200, C $300, and D $500.

3. What number is that which being increased by its ½, its ¼, and 16 more, will be doubled? *Ans.* 64.

4. D, E and F, playing at cards staked 324 crowns, but disputing about tricks, each one took as many as he could get. D got a certain number, E got as many as D and 15 more, and F got ⅕ part of both their sums added together; how many did each get? *Ans.* D got 127½, E 142½, and F 54.

5. A man has 100 acres of land in 3 lots. The second lot contains twice as much as the first, lacking 8 acres, and the third contains three times as much as the first, lacking 15 acres; how many acres does each lot contain?

Ans. the 1st contains 20½ acres, the 2d 33, the 3d 46½a.

6. A and B laid out equal sums of money in trade; A gained $150, and B lost $225; then A's money was double that of B's. What sum did each lay out?

1st, Supp.	A $300	B $300
	+150	−225
	450	75
double $75	=150	
1st error	=300	

2d Supp.	A $400	B $400
	+150	−225
	550	175
double $175	=350	
2d error	=200	

Ans. Each laid out 600 dollars.

7. A man agreed to work 70 days on this condition, that for every day he worked he should receive $,80, and for every day he was idle he should pay $,30; and at the expiration of the time he received $31,80. How many days did he work and how many was he idle?

Ans. he worked 48 days, and was idle 22.

8. A farmer having driven his cattle to market received for them all $546, being paid for every ox $30, for every cow $20, and for every calf 3½. There were twice as many cows as oxen, and three times as many calves as cows; how many had he of each?

Ans. 6 oxen 12 cows 36 calves.

9. A man gave his estate to his three sons in the following manner, viz.: to B he gave half, lacking $130, to C one-third, and to D the rest, which was $75 less than the share of C; what was the amount of the whole estate, and how much was each one's portion?

Ans. The whole amount was $1230, and
B had $485, C $410, and D $335.

10. Three men are to share a certain sum of money, as follows, viz.: the first is to have twice as much as the third, and the second two-thirds as much as the first, and the shares of the second and third, added together, are $1435; what is the share of each?

Ans. The first $1230, the second $820, the third $615.

Questions.

1. What is Position?
2. How is it divided?
3. What is Single Position?
4. What is the Rule for Single Position?
5. What is Double Position?
6. In Single Position, how is the number sought always multiplied or divided?
7. How in Double Position?
8. What is the Rule for Double Position?

DUODECIMALS.

Duodecimals are fractions of a foot. They are so called from the Latin word *duodecim*, which signifies twelve. The foot being divided into 12 equal parts, called inches, or primes, and each of these parts again divided into 12 other equal parts, called seconds, and each second again

divided into twelve other equal parts called thirds, and each third into twelve equal parts called fourths; and so on to any extent.

Thus, 1 inch or prime is - - $\frac{1}{12}$ of a foot.
1 second is $\frac{1}{12}$ of $\frac{1}{12}$ - - - $\frac{1}{144}$ of a foot.
1 third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - - $\frac{1}{1728}$ of a foot.
1 fourth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ - $\frac{1}{20736}$ of a foot, &c.

It is usual to distinguish inches by one mark, thus (′) seconds by two marks, (″) thirds by 3 marks, (‴) fourths by 4 marks, (⁗) &c. These marks are called indices.

Twelve of each of the less denominations are equal to one of the next greater, as in the following

TABLE.

12⁗ fourths make	-	1‴ third.
12‴ thirds make	- - - -	1″ second.
12″ seconds make	- -	1′ inch or prime.
12′ inches or primes make	-	1 foot.

Hence they are added and subtracted in the same manner as compound numbers.

MULTIPLICATION OF DUODECIMALS

Is used in finding the contents of surfaces and solids.

(*Note.* F stands for feet, and I or ′ for inches.)

F×F give - - - -	F	I×I give - - - - -	″
F×I, or I×F, give -	I	I×″, or ″×I, give -	‴
F×″, or ″×F, give -	″	″×″ give - - - -	⁗

That is, the product of any two denominations will always be of that denomination denoted by the sum of their indices.

RULE.

1. Write the multiplier under the corresponding denominations of the multiplicand.

2. Multiply each denomination in the multiplicand by the highest denomination in the multiplier, carrying 1 to the next higher for every 12 in the lower denomination.

3. Multiply each denomination in the multiplicand by the second denomination in the multiplier, and set the result of each denomination one place removed to the right of the

former products; and so on for each of the other denominations of the multiplier, always placing the product by a smaller denomination one place farther to the right than that of its superior denomination.

EXAMPLES

1. How many square feet are contained in a board 6 feet 7 inches long, and 2 feet 5 inches wide?

Operation.

F	′	
6	7	
2	5	
13	2′	
2	8′	11″
15	10′	11″

Illustration.—We first multiply the multiplicand, beginning with the inches, by the two feet in the multiplier; thus, $7 \times 2F = 14' = 1F$ 2in. We place the 2′ under the inches, and carry the 1 to the feet; thus, $6F \times 2F = 12$, and 1 to carry $= 13F$. We then multiply by the 5′ in the multiplier; thus, $7' \times 5' = 35'' = 2'\ 11''$; we set the 11″ one place to the right, and carry the 2′ to the next; thus, $6F \times 5' = 30'$, and 2′ to carry make $32' = 2F\ 8'$, which we set down in their proper places, and the products added together give the answer, 15F 10′ 11″, or 15F 10$\frac{11}{12}$ inches.

2. How many square feet are contained in a board 15 feet 5 inches long, and 1 foot 8 inches wide?

	15F	5′	
	1	8′	
	15	5′	
	10	3′	4″
Ans.	25	8′	4″

$5' \times 1ft. = 5'$, and $15ft. \times 1ft. = 15ft.$ Then, $5' \times 8' = 40'' = 3'4''$, and $15ft. \times 8' = 120'$, and 3′ to carry make $123' =$ 10 feet 3′.

3. How many solid feet in a block 2ft. 6 inches long, 1ft. 9 inches wide, and 1ft. 5 inches thick?

	2ft.	6′ length			
	1	9 breadth			
	2	6′			
	1	10′	6″		
	4	4′	6″		
	1	5′			thickness
	4	4′	6″		
	1	9′	10″	6‴	
Ans.	6	2′	4″	6‴	

Note. The length multiplied by the breadth, and that product by the thickness, gives the cubic, or solid contents.

4. How many square feet are contained in a floor 16ft. 9 inches long, and 12ft. 4 inches wide? *Ans*. 206ft. 7in.

5. How much wood in a pile 6ft. long, 3ft. 9in. high, and 4ft. 6in. wide? *Ans*. 101ft. 3in.

6. How many square feet are contained in 15 boards, each 18ft. 5in. long, and 1ft. 7in. wide? *Ans*. 437f. 4′ 9″.

First find the square contents in 1 board which multiply by 15, using the factors.

7. Multiply 16ft. 9in. by 7ft. 11in. *Ans*. 132ft. $7\frac{3}{12}$in.

8. Multiply 10ft. 4′ 3″ by 2ft. 8′. *Ans*. 27ft. 7′ 4.

9. How many square feet in a board 36ft. 8in. long, and 2ft. 4′ 9″ wide? *Ans*. 87ft. 10′ 2″.

10. How much wood in a load 8ft. long, 4ft. 5′ high, and 3ft. 9′ wide? *Ans*. 132ft. 6′=1 cord, 4 $\frac{1}{2}$ft

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MISCELLANEOUS QUESTIONS FOR EXERCISE IN THE FOREGOING RULES.

1. What is the sum of $1948\frac{1}{4}$ added to itself? *Ans*. $3896\frac{1}{2}$.

2. There are three numbers, their sum is 2302, the first is 311, and the second 695, what is the third? *Ans*. 1296.

3. What is the difference between 31 eagles, and 3099 dimes? *Ans*. 10cts.

4. If the minuend be 3441, and the remainder 365, what is the subtrahend? *Ans*. 3806.

5. What number being multiplied by 5, will make just as much as 25 multiplied by 49? *Ans*. 245.

6. What number is that which being divided by 96 the quotient will be 128? *Ans*. 12288.

7. There are 6 chests of drawers, each containing 18 drawers—in each drawer is 6 divisions, and each division contains $36. How many dollars in all? *Ans*. $23328.

A line or vinculum drawn over several numbers, denotes that the numbers under it are to be taken jointly, or as one whole number.

8. $\overline{5+8}\times\overline{9-2}=91$.

9. $\overline{12-3+5}\times\overline{8+5-7}=$How many? *Ans*. 84.

10. $\overline{40+7-15}\times\overline{12-3+7}=$How many? *Ans*. 512.

11. A man being asked his age replied, I have 7 sons, the difference between whose ages is just 2 years—I was 34 years old when my oldest son was born, and that is now the age of my youngest. What was his age?

Ans. 80 years.

12. If from a staff 8 feet in length,
A shadow, 5 is made,
What is that steeple's height in yards,
That's 90 feet in shade? *Ans.* 48 yards.

13. What number, being multiplied by 15, the product will be $\frac{3}{4}$? *Ans.* $\frac{3}{4} \div \frac{15}{1} = \frac{1}{20}$.

14. What number is that which being divided by 15, the product will be $\frac{3}{4}$? *Ans.* $\frac{3}{4} \times 15 = 11\frac{1}{4}$.

15. What number is that, to which if you add $\frac{1}{3}$, will be $\frac{5}{8}$? *Ans.* $\frac{5}{8} - \frac{1}{3} = \frac{7}{24}$.

16. What number is that, from which if you subtract $\frac{2}{3}$, the remainder will be $\frac{2}{15}$? *Ans.* $\frac{4}{5}$.

17. B and C traded together and gained \$100; B put in \$640, C put in so much that he must receive \$60 of the gain. How much did C put in? *Ans.* \$960.

18. Divide \$356 between B and C, so that the shares shall be in proportion to each other as 3 to 5.

$$3+5=8 : 356; \left\{ \begin{array}{l} 3 : \$133\tfrac{1}{2} \text{ B's part.} \\ 5 : \ \ 222\tfrac{1}{2} \text{ C's part.} \end{array} \right.$$

19. Bought cloth at \$1,25 per yard, and lost 25 per cent by the sale of it. How was it sold per yard?

Ans. \$93,7 $\frac{5}{10}$m.

20. Thomas sold 150 pine apples at $33\frac{1}{3}$cts. apiece, and received as much money as Harry received for a certain number of watermelons which he sold for 25 cents apiece. How much money did each receive, and how many melons had Harry?

Ans. each received \$50, and Harry had 200 melons.

21. B and C depart from the same place and travel in opposite directions, B goes east 23 miles a day, and C travels west 35 miles a day. How far will they be apart at the end of 6 days, and how many miles will each have traveled?

Ans. 348 miles. B will have traveled 138m. and C 210.

22. A certain pasture will last 936 sheep 7 weeks.—

How many must be turned out, so that it will be sufficient to last the remainder 9 weeks? *Ans.* 214.

23. A merchant bought a quantity of flour for $137, and sold it again for $143, what did he gain per cent?

Ans. $4\frac{52}{137}$ per cent.

24. Said Henry to Richard, my purse and money are worth $43,75. But the money is thirty-four times as much as the purse. I demand how much the purse was worth, and how much money it contained?

Ans. $1,25 and it contained $42,50.

25. A young man received £210, which was $\frac{3}{4}$ of his elder brother's portion, now three times the elder brother's portion was half the father's estate. What was the value of the whole estate? *Ans.* £1890

26. What is the interest of $256 from January 1st, 1833, to Sept. 16th, 1835, at 6 per cent? *Ans.* 41,60.

27. A note dated April 15th, 1828, for $756,20 was paid July 10th, 1836. What was the amount paid, interest being computed at 6 per cent per annum?

Ans. $1129,88c. 8m.+

28. What is the difference between the interest of $500 at 5 per cent for 12 years, and the discount of the same sum at the same rate and for the same time?

Ans. $112,50.

29. What is the yearly insurance of a Cotton Factory, valued at $35640, at $2\frac{5}{8}$ per cent premium? *Ans.* 935,55.

30. What is the cost of $1500 Bank Stock, at $8\frac{1}{3}$ per cent advance, or at $108\frac{1}{3}$ per cent? *Ans.* $1625.

31. Suppose 2 men start from the same place and travel in opposite directions, one at the rate of 5 miles an hour, and the other $\frac{2}{3}$ as fast. How far apart would they be at the end of 13 hours? *Ans.* $108\frac{1}{3}$ miles.

32. A hare starts 40 yards before a grey hound, and is not perceived by him until she has been up 40 seconds; she scuds away at the rate of 10 miles an hour, and the dog makes after her at the rate of 18 miles an hour. How long will the course hold, and what space of ground will be run over from the place where the dog started?

Ans. $60\frac{5}{22}$sec. and 530yds. space.

33. Three persons purchase a sloop, towards the pay

ment of which A advances $\frac{3}{8}$, B $\frac{3}{7}$, and C £140. How much paid A and B, and what part of the vessel had C?

$\frac{3}{8}$ and $\frac{3}{7}$ reduced to a common denominator become $\frac{21}{56}$ and $\frac{24}{56}$, and $\frac{21}{56}+\frac{24}{56}=\frac{45}{56}$; then the whole $\frac{56}{56}-\frac{45}{56}=\frac{11}{56}$ C's part of the vessel. Then as $\frac{11}{56}$: £140 : : $\frac{21}{56}$ to £$267\frac{3}{11}$ A paid, and $\frac{11}{56}$: 140 : : $\frac{24}{56}$ to £$305\frac{5}{11}$ B paid.

34. The third part of an army was killed, the fourth taken prisoners, and 1000 fled. How many were in this army? How many killed? and how many captives?*

$\frac{1}{3}+\frac{1}{4}=\frac{7}{12}$ of the army, and $\frac{12}{12}-\frac{7}{12}=\frac{5}{12}$, therefore, 1000 is $\frac{5}{12}$ of the army; then if $\frac{5}{12}$ be 1000, how many is $\frac{12}{12}$, or the whole?

Ans. 2400 in the army, 800 killed, and 600 prisoners.

35. There is a mast or pole which stands $\frac{1}{6}$ of its length in the mud, 12 feet of it in the water, and $\frac{5}{6}$ of its length in the air, or above water, what is its whole length?*

Ans. 216ft.

36. What number is that which being divided by $\frac{3}{4}$ the quotient will be 21? *Ans.* $15\frac{3}{4}$.

37. In an orchard of fruit trees, $\frac{1}{2}$ of them bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ peaches, and 50 of them cherries. How many trees does the orchard contain? *Ans.* 600

38. There is a certain number which being divided by 7, the quotient resulting multiplied by 3, that product divided by 5, from the quotient 20 being subtracted, and 30 added to the remainder, the half sum shall make 35. Can you tell me the number? *Ans.* 700.

39. A farmer being asked how many sheep he had, answered that he had them in 5 fields—in the first he had $\frac{1}{4}$ of his flock, in the second $\frac{1}{6}$, in the third $\frac{1}{8}$, in the fourth $\frac{1}{12}$, and in the fifth 450. How many had he?* *Ans.* 1200.

40. A gentleman divided his fortune among his 3 sons, giving A \$9 as often as B \$5, and to C but \$8 as often as B \$7, and yet C's dividend was \$2584. What was the whole amount of the estate? *Ans.* 19466,13+

41. A and B together, can build a boat in 18 days, and

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* This and all those questions marked with an asterisk, may be solved by the rules of Position, or on general principles by fractions. And the scholar may, for practice, solve them by both methods. But he should be required to pay particular attention to the method of solving by fractions, which is generally much preferable to that of position or supposition.

with the assistance of C, they can do it in 11 days. In what time would C do it alone? *Ans.* $28\frac{2}{7}$d.

42. A and B laid out equal sums of money in trade—A gained a sum equal to $\frac{1}{4}$ of his stock, and B lost \$225.—Then A's money was double that of B's. What sum did each lay out?*

Thus the whole $=\frac{4}{4}$, and A gained $\frac{1}{4}+\frac{4}{4}=\frac{5}{4}$, the amount of A's stock and gain together, which is double B's; then $\frac{1}{2}$ of $\frac{5}{4}=\frac{5}{8}$, and $\frac{5}{8}$ taken from $\frac{8}{8}$ leaves $\frac{3}{8}$, which B lost.—Then as $\frac{3}{8}$: \$225 :: $\frac{8}{8}$ to the answer.

Ans. each laid out \$600.

43. If to my age there added be,
One half, one third, and three times three,
Six score and ten the sum will be,
What is my age? pray tell it me.*

$\frac{1}{2}+\frac{1}{3}=\frac{5}{6}$, and the whole $\frac{6}{6}+\frac{5}{6}=\frac{11}{6}$. Therefore, $\frac{11}{6}=130-9$. Then $\frac{11}{6}$: 121 :: $\frac{6}{6}$: *Ans.* 66.

44. If A can do a piece of work in 20 days, B in 30, and C in 60 days, in what time will they all do it by working together?

days. work. days.

$$\left.\begin{array}{l} 20 : 1 :: 1 : \frac{1}{20}\text{A} \\ 30 : 1 :: 1 : \frac{1}{30}\text{B} \\ 60 : 1 :: 1 : \frac{1}{60}\text{C} \end{array}\right\}$$

and $\frac{1}{20}+\frac{1}{30}+\frac{1}{60}=\frac{6}{60}$. Therefore, A B and C can do $\frac{6}{60}$ of it in 1 day; as $\frac{6}{60}$: 1 day :: $\frac{60}{60}$. *Ans.* 10 days.

45. What number is that which being increased by its $\frac{1}{2}$, its $\frac{1}{4}$, and 18 more, will be doubled?*

$\frac{1}{2}+\frac{1}{4}=\frac{3}{4}$ and $1=\frac{4}{4}+\frac{3}{4}=\frac{7}{4}$ and double $\frac{4}{4}=\frac{8}{4}$ and $\frac{8}{4}-\frac{7}{4}=\frac{1}{4}$, consequently 18 is $\frac{1}{4}$ of the required number. *Ans.* 72.

46. What number is that to which if $\frac{1}{7}$ of itself be added the sum will be 40?* 40 is $\frac{8}{7}$ of what number? *Ans.* 35.

47. What number is that, which being increased by its $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, and 14 more will be made 3 times as large?*

$14=\frac{1}{12}$ of the required number. *Ans.* 168.

48. A Stationer sold quills at 11s. per thousand, by which he cleared $\frac{3}{8}$ of the money; but growing scarce he raised them to 13s. 6d. per thousand; what might he clear per cent by the latter price? *Ans.* £96 7s. $3\frac{3}{11}$d.

49. The hour and minute hand of a watch are exactly together at 12 o'clock. When are they next together?

The minute hand passes over 12 spaces while the hour hand passes over 1 space, consequently the minute hand

gains upon the hour hand 11 spaces in 1 hour; and it must gain 12 spaces in order to coincide with it.

Therefore, 11 : 1 : : 12 *Ans.* 1h. 5m. $27\frac{3}{11}$s.

Then if we multiply the above answer by 2, 3, 4, &c., it will show the times when the hands are next together.

50. A person being asked what o'clock it was, replied that it was between 3 and 4, and that the hands on his watch were exactly together. I demand how much it was past 3 o'clock? *Ans.* 16m. $21\frac{9}{11}$s.

51. A gentleman left his son a fortune, $\frac{1}{4}$ of which he spent in three months; $\frac{3}{7}$ of the remainder lasted him 8 months longer, when he had only $2850, left. Pray what did his father bequeath him? *Ans.* $6650.

52. A father left his two sons (the one 11, and the other 16 years old,) 10000 dollars, to be divided so that each share being put at interest at 5 per cent might amount to equal sums when they would be respectively 21 years of age. Required the shares. *Ans.* $\$5454\frac{6}{11}$, and $\$4545\frac{5}{11}$.

53. A lets B have 120 gallons of brandy, worth 95cts. for $1,25 per gallon, $\frac{1}{4}$ of which B is to pay in cash. B has paper worth $2 per ream, which he gives A for the rest of his brandy, at $2,50 per ream. Which gets the best of the bargain, and how much? *Ans.* A by 13,50.

54. Divide $600 among 3 men, in such a manner that as often as the first has $3, the second shall have $5, and the third $7. How many dollars will each receive? (See ex. 18.) *Ans.* A 120, B 200, and C 280.

55. A gentleman had £7 17s. 6d. to pay among his laborers—to every boy he gave 6d., to every woman 8d., and to every man 16d., and there were for every boy, 3 women, and for every woman, 2 men. I demand the number of each? *Ans.* 15 boys, 45 women, and 90 men.

56. Bought a quantity of broadcloth for $2070, and if the number of dollars which it cost per yard, were added to the number of yards bought, the sum would be 351. What was the number of yards bought, and how many dollars was it per yard? (Solved by Prob. 6, page 188.)

Ans. 345yds. at $6 per yard.

57. Bought a quantity of cloth for £383 5s. and the difference between the number of shilling it cost per yard, and

the number of yards bought, was 344. I demand the number of yards bought, and the price it cost per yard. (Solved by Prob. 7, page 189.)

Ans. 365 yards, at 21s. per yard.

58. A farmer driving his sheep to market, was met by a person who inquired how many sheep he had; to avoid a direct answer, the farmer replied, if you multiply one-half and three-fourths of my number together, the product will be 864. Can you tell me how many sheep he had? (Solved by Prob. 1, page 194. *Ans.* he had 48.

59. Divide \$500 between A and B so that A's share shall be $\frac{3}{7}$ as much as B's.*

$$B\ \tfrac{7}{7}+A\ \tfrac{3}{7}=\tfrac{10}{7}\left\{\begin{array}{l}\tfrac{10}{7}:\$500::\tfrac{7}{7}:\$350\text{ B's.}\\ \tfrac{10}{7}:\$500::\tfrac{3}{7}:\$150\text{ A's.}\end{array}\right.$$

60. What o'clock is it in the afternoon, when the time past from noon is equal to $\frac{2}{13}$ of the time to midnight.*

$\frac{13}{13}+\frac{2}{13}=\frac{15}{13}$, $\frac{15}{13}:12::\frac{2}{13}:$ *Ans.* 36 minutes past 1.

61. A and B talking of their ages, A said that $\frac{2}{3}$ of his age was equal to $\frac{3}{4}$ of B's, and that the sum of their ages was 68. Required the age of each.

(Reduce the fraction to a common denominator, and the numerator will be the proportions, ex. 18.)

Ans. A's 36, B's 32.

62. William and Henry enter into partnership, and buy a stock of goods, and at the end of 12 months, having sold the goods, they find that they have gained at the rate of 200 per cent upon the prime cost; they then divide their gain between them in proportion to the purchase money paid by each, which was as 5 to 7; and Henry says to William, my part of the gain is really a handsome sum of money, and if I had as many such sums as your part contains dollars, I should then have \$686000. Required the sum of money paid by each in purchasing the stock. (Solved by Prob. 2, page 195.)

Ans. William paid \$350, and Henry \$490.

AN

APPENDIX;

CONTAINING

USEFUL PROBLEMS IN THE MENSURATION OF SUPERFICES AND SOLIDS.

SECTION I.—SUPERFICIES.

The area of every plain surface is conceived to be made up of a certain number of squares, either greater or less, according to the measure by which the dimensions are taken, which is generally in inches, feet, yards, rods, &c. A square inch means a space an inch long and an inch broad, in which depth or thickness is not considered; and so of square feet, yards, rods, &c. And the superficial contents, or area of any plain surface, is the number of square inches, feet, yards, rods, acres, &c., which it contains.

PROBLEM I.—To find the area of a Square.

RULE.

Multiply the side of the square into itself, and the product will be the area, or contents.

EXAMPLES.

1. How many square feet of boards are contained in the floor of a room which is 16 feet square? Ans. 256.

2. How many acres are contained in a square field which measures 65 rods on each side? [Reduce the whole number of square rods to acres.] Ans. 26 acres 1 r. 25 rods.

PROB. II.—To find the area of a parallelogram, or long square.

RULE.

Multiply the length by the breadth, and the product will be the area.

EXAMPLES.

1. How many square yards of ground are contained in a garden which is 126 feet long and 65 feet wide? [9 sq. feet=1 sq. yard, therefore ÷ the sq. feet by 9.] Ans. 910 sq. yds.

2. How many acres are contained in a lot of land in the form of a long square, which is fifty-six rods in length and thirty-seven rods in width? Ans. 12 acres 3 r. 32 rods.

3. How many feet in a board or plank 18 feet long, and 1 foot 6 inches wide?

By duodecimals, 18F 0′×1F 6′=27ft. 0′, Ans.

By decimals, 1ft. 6in.=1,5ft. Then 18×1,5=27, Ans

Or, multiply the length in feet by the breadth in inches, and divide the product by 12. Thus, 18ft.×18in.÷12=27ft. Ans. as before.

4. How many square feet are contained in a piece of board 126 inches long and 16 inches wide?

Note.—144 square inches, $=12\times12$, make 1 square foot; therefore divide the inches by 144. Ans. 14 sq. feet.

5. How much length, that is 9 inches wide, will make a square foot? 1 sq. ft.$=$144 sq. in.; therefore $144\div9=16$ inches, Ans.

6. How many rods in length must a piece of land be, which is 5 rods wide, to make an acre?

1 acre$=$160 square rods, then $160\div5=32$ rods, Ans.

7. There is a piece of land in the form of a long square, which contains 6 acres. Its length is 120 rods; I demand its width?

6a.$=$960 sq. rods. Then $960\div120=8$ rods, Answer.

Prob. III.—To measure a Triangle.

A triangle is any three cornered figure, which is bounded by 3 right lines.

Triangles are either right-angled or oblique,

FIGURE 1.
Right-angled Triangle.

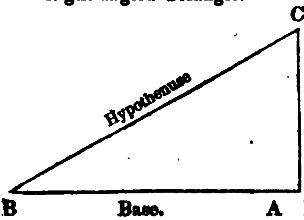
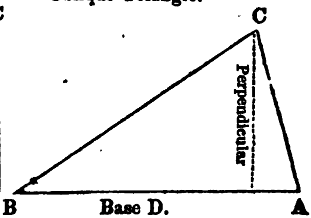


FIGURE 2.
Oblique Triangle.



A right-angled triangle is that which has one right angle, that is, square corner, as the angle A, Fig. 1; in which the side A B is called the base, the side A C the perpendicular, and the side B C the hypothenuse. Note. Both the base and perpendicular of a right-angled triangle are sometimes called the legs.

Art. 1.—To find the area of a Triangle, (either right-angled or oblique.)

RULE.

Multiply the base by half the perpendicular, or multiply the perpendicular by half the base, and the product will be the area.

Or, multiply the whole base by the whole perpendicular, and one half the product will be the area.

1. What is the area or contents of a triangle whose base is 36 feet, and perpendicular 24 feet? $36\times12=432$ sq. feet, Ans.

2. In a triangular lot of land, whose base measures 51½ rods and perpendicular 48 rods, how many acres? Ans. 7a. 2r. 36 rods.

3. What is the area of a triangular field which measures on the base 65 rods, and the perpendicular from the corner opposite the base, to the base, is 27 rods? $65\times13,5=877,5$r.$=$5a. 1r. 37½ rods.

Art. 2.—In every right-angled triangle, the square of the hypothenuse is equal to the sum of the squares of the other two sides.

1. Hence, when the legs are given, to find the hypothenuse.

RULE.

Add the squares of the two legs together, and extract the square root of their sum.

2. When the hypothenuse and one leg is given, to find the other leg.

RULE.

From the square of the hypothenuse subtract the square of the given leg, and the square root of the remainder will be the other leg.

1. There is an edifice whose height is 50 feet, and the width of the street running by it is 36 feet. What is the length of a ladder that will reach from the opposite side of the street to the top of the edifice?

$50^2=2500$, and $36^2=1296$.

Then $\sqrt{2500+1296}$ Ans. 61,6ft.+

2. Suppose the foot of a ladder, which is 32 feet in length, being placed on a level 16 feet from the bottom of a building, will just reach the top of the same; what is the height of the building?

(In this example, the length of the ladder is the hypothenuse.)

$\sqrt{32\times32-16\times16}=27,7$ft.+ Ans.

3. Two ships sail from the same port, one due east 60 miles, and the other due south 40 miles; how far are they apart?

(In this example, the legs are given to find the hypothenuse.)

Answer, 72,1+miles.

Prob. IV.—To measure Circles.

Art. 1. The diameter of a circle given, to find the circumference.

Note.—The diameter is a right line drawn across the centre of a circle, dividing it into two equal parts.

RULE.

As 7 is to 22, or more accurately, as 1 is to 3,14159, so is the diameter to the circumference.

Art. 2. The circumference given, to find the diameter.

RULE.

As 22 is to 7, or as 3,14159 is to 1, so is the circumference to the diameter.

EXAMPLES.

1. What is the circumference of a wheel or circle whose diameter is 5 feet? 7 : 22 :: 5 .... Ans. 15,7+feet.

2. What is the diameter of a circle whose circumference is 110 feet? 22 : 7 :: 110 .... Ans. 35.

Art. 3. To find the area of a circle.

RULE.

Multiply half the diameter by half the circumference, and the product will be the area. Or, multiply the square of the diameter by .7854, and the product will be the area.

EXAMPLES.

1. Required the area of a circle whose diameter is 21 inches, and circumference 66 inches?

½ circumference 66=33 inches, and ½ diameter 21=10,5 inches; Then, 33×10,5=346,5. Ans. 346,5 inches.

2. What is the area or contents of a circle whose diameter is 20 rods? [By the second method.] 20×20=400, square of the diameter. Then 400×,7854=314,16 rods = 1 acre 3r. 34,16 rods.

3. If the area of a circle be 78,54 sq. feet, what is the diameter?

Note.—The area, divided by ,7854, will show the square of the diameter; then the square root of that will be the diameter required. Thus, 78,54÷,7854=√100=10 feet, diameter.

ART. 4. To find the area of a globe or ball.

The area of the surface of a globe or ball is 4 times as much as the area of a circle of the same diameter.

Therefore, multiply the whole circumference into the whole diameter, and the product will be the area.

1. How many square inches of paper will it take to cover a globe that is 12 inches in diameter?

7 : 22 :: 12 : the circumference, 37,7+ inches. Then 37,7×12 =452,4 sq. in., and 452,4÷144=3,14 sq. feet.

SECTION II. SOLIDS.

A solid body is that which has length, breadth, and thickness. Solids are generally estimated by the solid inch, solid foot, &c.

PROB. I.—To find the solidity of a Cube.

A cube is a solid having six equal sides, each of which is an exact square.

RULE.

Multiply the side by itself, and that product by the same side, gives the solid contents.

1. How many solid inches are contained in a cube 12 inches (=1 foot) long, 12 inches wide, and 12 inches thick?

12×12×12=1728 solid inches, which are equal to 1 solid foot.

2. Suppose a cellar to be dug which shall be 11 feet 6 inches every way, in length, breadth and depth; how many solid feet of earth must be thrown out of the same?

11ft. 6in.=11,5ft. then 11,5×11,5×11,5=1520,875 solid ft. Ans.

PROB. II.—To find the contents of any regular solid of three dimensions, length, breadth and thickness, as a piece of timber squared, whose length is more than its breadth and depth.

RULE.

Multiply the length, breadth and thickness continually together, and the product will be the contents.

EXAMPLES.

1. If a square piece of timber be 1 foot 6 inches broad and 9 inches thick, and 8 feet long, how many solid feet does it contain?

1. By decimals.

1ft. 6 in.=	1,5ft.
9in.=	,75ft.
	75
	105
	1,125
length=	×8
Ans,	9,000=9 solid ft.

2. By duodecimals.

	F	′	″
breadth =	1	6	
thickness=	×	9	
	1	1′	6″
length	×8	0	0
Ans.	9	0	0

Or, 1ft. 6in.=18in., and 8ft.=96in.; then 18in.×9in.×96in.=15552 solid inches; and 15552÷1728=9 solid feet, Answer.

Note.—The breadth in inches, muliplied by the depth in inches, and that product multiplied by the length in feet, and the last product divided by 144, will give the solid contents in feet, &c.

2. What are the solid contents of a stick of timber which is 22 feet long, 15 inches broad, and 9 inches thick?

in. in. ft.
Thus, 15×9×22=2970÷144=20,625ft., Ans.

3. How many solid feet are contained in a piece of timber 14in. broad, 9 inches thick, and 8 feet long? Ans. 7 feet.

Prob. III.—The breadth and thickness of a piece of timber given in inches. to find how much in length will make a solid foot.

RULE.

Divide 1728, (the number of inches in a solid foot,) by the product of the breadth and depth, the quotient will be the length making a solid foot

EXAMPLES.

1. If a piece of timber be 14 inches broad and 9 inches deep, how much length will make a solid foot?

Thus, 14×9=126, and 1728÷126=13,7+ inches, Ans.

2. If a piece of timber be 18 inches broad and 14 inches deep, how much length will make a solid foot?

18×14=252; then 1728÷252=6,85+ inches, Ans.

Prob. IV.—To measure a Cylinder.

A cylinder is a round body, whose bases are circles like a round column or stick of timber, of equal bigness from end to end.

RULE.

Multiply the square of the diameter of the end by ,7854, which gives the area of the base; then multiply the area of the base by the length, and the product will be the solid contents. Or,

Note.—When the circumference and diameter both are given, you may multiply half the circumference by half the diameter, and that product multiplied by the length, will give the solid contents.

EXAMPLES.

1. What is the solid contents of a round stick of timber of equal bigness from end to end, whose diameter is 21 inches, (=1ft. 9in.) and length 20 feet?

Thus, 1 foot 9 inches=1,75ft.
×1,75

square of diameter =3,0625×,7854=2,4052+ area of the base.
20

Answer, solid contents =48,1040+ feet.

Or, 21 in.×21in.×,7854=346,3614in. area of the base.
20=length in feet.

144)6927,2280(48,106 nearly.

Prob. v.—To measure Pyramids and Cones, or the frustrum of a Pyramid or Cone.

Solids which decrease gradually from the base until they end in a point are called pyramids or cones. If the base be square, it is called a square pyramid. If the base be a triangle, it is called a triangular pyramid.—But if the base be round, it is called a cone. The top is called the vertex; and a perpendicular line from the vertex to the base is called the perpendicular height.

Art. 1. The solid contents of any pyramid or cone may be found by multiplying the area of the base by $\frac{1}{3}$ of its perpendicular height.

The frustrum of a pyramid or cone is what remains after the top is cut off, by a plane parallel to the base.

Art. 2. To find the solid contents of the frustrum of a square pyramid, or tapering stick of square timber.

RULE.

Multiply the side of the greater base or end, by the side of the lesser base or end, and to the product add $\frac{1}{3}$ of the square of the difference of the sides of the bases, or ends; then multiply this sum by the perpendicular height or length.

EXAMPLE.

There is a tapering, square stick of timber, the side of the greater base or end of which measures 18 inches, the side of the lesser end 12 inches, and its length 30 feet; what is its solid contents?

Thus, greater end	18 inches.	18
less	do.×12 inches.	—12
product	=216	6
$\frac{1}{3}$ square of diff.	+12	×6
	228	3)36=square of the diff.
	30 length in feet.	
		12=$\frac{1}{3}$ of square of do.

144)6840(47,5 feet Answer.

Art. 3. To find the solid contents of the frustrum of a cone, or tapering stick of round timber.

RULE.

Multiply the diameters of the bases or ends together, and to the product add one third of the square of the difference of the diameters; then multiplying this sum by ,7854 gives the mean area between the two bases or ends, which multiplied by the length gives the solid contents.*

EXAMPLE.

If the diameter of one end of a tapering stick of round timber be 21 inches, and the diameter of the other end 12 inches, and the length 30 feet, what is the solid contents?

Thus, diameter of greater end	21		21
" " less end	$\times 12$		12
		diff. of diameters	9
	252		$\times 9$
$\frac{1}{3}$ sq. of the diff. of the diameters	$+27$	square of diff.	81
	279	$\frac{1}{3}$ of do.	27

Then, $279 \times ,7854 \times 30 \div 144 = 45,65+$ solid feet, Ans.

Prob. vi.—To find how many solid feet a round stick of timber, of equal bigness from end to end, will contain when hewn square.

RULE.

Multiply twice the square of the semi-diameter in inches by the length in feet, and divide the product by 144, and the quotient will be the answer.

EXAMPLE.

If the diameter of a round stick of timber be 21 inches, and its length 20ft., how many solid feet will it contain when hewn square?

Diameter $21\text{in} \div 2 = 10,5\text{in}$. semi-diameter.

Then $10,5 \times 10,5 \times 2 \times 20 \div 144 = 30,625$ feet, Ans.

Prob. vii.—To find the solid contents of a globe or sphere.

RULE.

Multiplying the circumference and diameter together gives the area, which multiplied by one-sixth of the diameter gives the solid contents.

* This is a correct way of measuring round logs, or timber, which taper gradually from end to end. But the old English method of measuring round timber was, to girt the stick in the middle, and call one-fourth of this girt the side of a square equal to the circumference. This one-fourth part squared, and the square multiplied by the length, they called the solid contents, which was an erroneous method; for, the girt in the middle is not the mean between the ends, nor one-fourth of the girt equal to the side of a square of equal area with circumference; and from this old erroneous practice of measuring timber was introduced the custom of calling 40 feet of round timber and 50 feet of hewn timber a ton, for 40 feet of round timber, measured by this method, will actually make about 50 feet of hewn timber.—We suppose, that when timber is accurately measured, 40 feet of every kind should make a ton.

EXAMPLE.

What is the solid contents of a globe or ball whose diameter is 18 inches?

Thus, the diameter being 18 inches, the circumference is found to be 56,57+ inches. (See Rule, page 235.) Then the circumference, 56,57 inches, multiplied by the diameter, 18 inches, =1018,+ which multiplied by one-sixth of the diameter, (=3 inches) gives 3054 solid inches, =Ans. 1 solid foot 1326in.

Prob. viii.—To find how many bushels, or gallons, will be contained in a vessel of given dimensions, (whether it be cubic, cylindric, or globular.)

RULE.

If it be a cubic vessel, find its contents or capacity in inches, by Prob I. or II. p. 236. If it be cylindric, find its contents by Prob. IV. If it be globular, by Prob. VII. Then, as 2150,4 cubic inches make a bushel, therefore, dividing by 2150,4 will give the bushels; and divide by 231, the number of inches in a wine gallon, gives the number of wine gallons; or, divide by 282, and you will have the beer gallons.

EXAMPLES.

1. How many bushels, and how many wine gallons, will a cistern hold that is 3 feet long, 2 feet wide, and 2 feet deep?

Thus, 3 feet =36 inches, and 2 feet =24 inches.

Then 36×24×24=20736 cubic inches.

And 20736÷2150,4=9,64 bushels, Answer.

And 20736÷231=Ans. 89 wine gallons, 177in. rem.

How many wine gallons are contained in a cylindrical vessel whose diameter is 18 inches, and depth 12 inches?

Thus, 18×18×,7854×12, (by Prob. 4, page 237,)=3053,6352 cubic inches, and 3053,6352÷231=13,2+ gallons, Ans.

Prob. ix.—The dimensions of the walls of a brick building given, to find how many bricks are sufficient to build it.

RULE.

From the whole extent of the wall, measured round on the outside, subtract four times its thickness, and multiply the remainder by the height, and that product by the thickness of the wall, gives the solid contents of the whole, which multiplied by the number of bricks in a solid foot, gives the answer.

Note.—To find the number of bricks in a solid foot, multiply the length in inches by the breadth in inches, and that product by the thickness; then divide 1728 by this product.

EXAMPLES.

How many bricks, 8 inches long, 4 inches wide, and 2½ inches thick, will it take to build the walls of a house 40 feet long, 30 feet wide, and 20 feet high, the walls to be one foot thick?

$8 \times 4 \times 2\frac{1}{2} = 80$ solid inches in a brick; then $1728 \div 80 = 21,6$ bricks a solid foot.

Then	$40+40+30+30=140$
four times the thickness	-4
	136
height	$\times 20$
solid feet in the whole wall	2720
number of bricks in a solid foot	$\times 21,6$
Answer	58752 bricks.

Prob. x.—To find the tonnage of a Ship.

RULE.

Multiply the length of the keel by the breadth of the beam, and that product by the depth of the hold, and divide the product by 95, the quotient will be the tonnage. Note.—If the vessel be double-decked, half the breadth of the mainbeam is accounted the depth of the hold.

EXAMPLES.

What is the tonnage of a vessel 65 feet in length by the keel, the breadth of her beam 22 feet, and the depth of the hold 10ft. 6in,= 10,5ft.? Thus, $65 \times 22 \times 10,5 \div 95 =$ Ans. 158 tons, 5 rem.

Prob. xi.—To find the number of gallons, &c., that are contained in a vessel in the form of the frustrum of a cone, or a tub, whose top and bottom diameters are unequal.

RULE.

Find the cubic contents of the given vessel in inches, (by the rule, Prob. V. Art. 3, for finding the contents of the frustrum of a cone,) which divided by 231 will give the wine gallons, &c.

Ex. How many wine gallons are contained in a tub, whose bottom diameter is 27 inches, top diameter 36 inches, and depth 50 inches?

Thus, $36-27=9$ diff. $\times 9 = 81$, sq. of diff. $\div 3 = 27$, $\frac{1}{3}$ of sq. of diff. And top diam. $36 \times$ bottom diam. $27 = 972, +27$, $\frac{1}{3}$ sq. of diff. $= 999$. Then $999 \times ,7854 \times 50 = 39230,73$ cubic inches, which $\div$ by 231 gives 169,83 gallons, Ans.

Prob. xii.—To guage a cask, or to find how many gallons it will hold.

To guage a cask, you must measure the head diameter and the bung diameter, (taking the measure within the cask,) and the length of the cask, making allowance for the thickness of the heads; then take the difference between the head and bung diameter, and when the staves are about an ordinary curve, add about 6 tenths of the difference to the head diameter, which will reduce the cask to a cylinder; that is, it will give the mean diameter.* Then multiply the square of the mean diameter in inches by ,7854, and that product by the length in inches, (Prob. 4.) gives the cubic

* If the diameters of the heads are unequal, take their mean diameter, and if the staves are very much curved, take ,66 instead of six tenths; but if they be nearly straight, take ,55, &c.

contents in inches, which divided by 231, gives the wine gallons, and by 282 gives the beer gallons.

But as the square of the mean diameter is always to be multiplied by ,7854 and divided by 231 for wine gallons, we may contract the operation, and multiply by their quotient, $\left(\frac{,7854}{231}=,0034;\right)$ thus, for wine gals. multiply by 34, pointing off 4 figures for the decimals; and for ale or beer gals., $\left(\frac{,7854}{282}=,0028\right)$ by 28, pointing off four figures for decimals. Hence the following

RULE.

Multiply the square of the mean diameter by the length, and multiply this product by ,0034 for wine, and by ,0028 for beer gallons.

Ex. There is a cask whose head diameter is 25 inches, bung diameter 30 inches, and whose length is 38 inches. How many wine gallons, and how many beer gallons does it contain?

Thus, the diff. between the head and bung diameter is 5 inches; this multiplied by 6 tenths gives 3in. to be added to the head diameter.

$25+3=28$in. mean diam. $\times 28$in. $=784$,sq. of mean diam. $\times 38$in. length, $=29792$. Then $29792\times ,0034=101,2928$ wine gals.; and $29792\times ,0028=83,4176$ beer gals.

PROB. XIII.—Of Mechanical Powers.

ART. 1. Of the Lever.—It is a fundamental principle in mechanics, that the power, and weight which will be raised, are to each other inversely as the spaces which they pass over. Hence to find what weight may be raised by a given power.

As the distance between the body to be raised, or balanced, and the fulcrum, or prop, is to the distance between the prop and the point where the power is applied, so is the power, to the weight which it will balance.

EXAMPLE.

1. There is a lever 10ft. long, and the fulcrum or prop, on which it turns, is 2ft. from one end; how many pounds weight at the end 2ft from the prop, will be balanced by a power of 56lbs. at the end 8ft from the prop? As 2ft. : 8ft. :: 56lbs. .... Ans. 224lbs.

2. What weight must be applied to the above lever eight feet from the prop, to balance one thousand pounds two feet from the prop? As 8ft. : 2ft. :: 1000lbs. .... Ans. 250lbs.

3. If 1000lbs. be placed 2 feet from the prop, on the above lever, how far must 250lbs. be placed from the prop on the other side, to balance the 1000lbs.? As 250lbs. : 1000lbs :: 2ft. .... Ans. 8ft.

ART. 2. Of the Wheel and Axle.—The spaces passed over are as their diameters or circumferences.

There is a windlass, the wheel of which is 60 inches in diameter, and the axis around which the rope coils is 6 inches in diameter; how many pounds on the axle will be balanced by 160lbs. at the wheel? As 6in. : 60in. :: 160lbs. .... Ans. 1600lbs.

2. How many pounds at the wheel of this windlass will balance 1200lbs. on the axle? As 60in. : 6in :: 1200lbs. .... Ans. 120lbs.

ART. 3. Of the Screw.—The power is to the weight which will be

raised, as the distance between the two threads of the screw, is to the circumference of a circle described by the end of the lever to which the power is applied.

EXAMPLE.

1. There is a screw whose threads are 1 inch apart. If it be turned by a lever 7 feet long, what weight will be balanced by 130 pounds power?

Thus, the lever is half the diameter, consequently the diameter is 14 feet. Then, as 7 : 22 :: 14 : 44, the circumference of the circle described$=$528 inches. Then, as 1in. : 528in. :: 130lbs., the ratio being as 528 to 1, &c. without any allowance for friction for which it is common to add about one-third to the power.

PROBLEM XIV.—To measure Loads of Wood.

Loads of wood are generally estimated by cord-feet, 8 of which make a cord: 128 solid or cubic feet make a cord; hence every 16 solid feet make 1 cord-foot.

RULE.

The length, breadth, and height, multiplied together will give the solid contents, which divided by 16, will give the cord-feet.

EXAMPLE.

How many cord-feet of wood in a load 8 feet long, 3 feet 6 inches wide, and 2 feet 6 inches high?

Thus, 3ft. 6in.$=$3,5ft.$\times$2ft. 6in.$=$2,5$=$8,75$\times$8$=$70,00 $\div$16$=$4$\frac{3}{8}$ cord-feet.

Or, by duodecimals, 3F 6′$\times$2F 6′$=$8F 9′$\times$8F 0′$=$70 solid feet. And 70$\div$16$=$4$\frac{3}{8}$, Answer.

But when the load is just 8 feet long, multiply the breadth and height together, and half the product will be the answer in cord-feet.

Thus, 3,5ft.$\times$3,5ft.$=$8,75$\div$2$=$4,375 cord-feet,$=$4$\frac{3}{8}$ft. the same as above.

A PRACTICAL

SYSTEM OF BOOK-KEEPING

FOR FARMERS, MECHANICS, AND MERCHANTS.

It is indispensably necessary, that every person who transacts any business should be acquainted with some concise, practical method of keeping his accounts. It will not be necessary for the pupil to understand every rule in arithmetic before he attends to this subject: every scholar should obtain some practical knowledge of Book-keeping, before he leaves school. There are various methods of keeping ac counts, and we have here inserted the two which we consider best adapted to common business.—The best method for Farmers and Mechanics, whose accounts are not extensive, is, to have a single book, and enter each person's name, with whom an

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BOOK-KEEPING, (METHOD FIRST.)

DR. TIMOTHY CAREFUL, Farmer.

			$	cts
1834				
May	10	To repairing cart wheels . . .	3	23
"	13	" one plough	6	50
June	7	" Frame for a barn . . .	50	00
Sept.	20	" 1 day, myself and apprentice, repairing house, &c.	2	50
1835				
Jan.	5	" window sashes for your house . .	11	58
April	25	" 7 days' work of myself on your house, ending this day, *a* $1,12 per day . .	7	84
"	"	" 7 do. do. of my apprentice, *a* $0,75 .	5	25
			86	90

Note.—When an account is settled and balanced, it is customary to draw parallel lines under it as above.

DR. JEREMIAH H. GOODALE.

			$	cts
1834				
Oct.	9	To 350 feet of boards, at 2½cts . .	8	75
Nov.	12	" 6 window blinds for your house, and hanging		
1835		the same	12	25
June	8	" 5 days' work of myself, repairing your house, laying floors, &c. at $1,08 per day .	5	40
"	"	" 5 do. do. of my apprentice, *a* 75cts. .	3	75

account is to be opened, on the left hand page, Dr., and opposite thereto, on the right hand page, Cr., (appropriating the whole or half of a page to each person's account, as the case may require,) writing down each entry at full length, and the dates against them, as in Method First. But where many entries are to be made daily, (as in the case with Merchants, &c.) it is necessary to have a DAY BOOK, and a LEGER, as exemplified in Method Second.

In the Day Book, the owner should charge every person Dr. to each article delivered them on account, and Cr. by each article received from them, at the time when the same was delivered or received; and every entry on the Day Book should be written at full length, and mention all the particulars necessary to make it fully understood.

Each entry made on the Day Book is to be posted, or placed in the Leger, assigning to each person's name a page, or part of a page, for the same; the Dr. being entered on the left, and the Cr. on the right hand of the page. When any article is posted, or transferred from the Day Book to the Leger, it should be noted on the margin of the Day Book, either by making a cross, (X) or two parallel lines, (||) and placing the figure opposite, denoting the page of the Leger to which it is transferred; and the date, and the page of the Day Book should also be placed on the Leger, to show the page of the Day Book from which each entry was taken; as exemplified in Method Second.

BOOK-KEEPING (METHOD FIRST.)

CONTRA. CR.

			$	cts
1834				
May	8	By your team one day ploughing	1	87½
	15	" 18¾lbs butter, *a* 14cts. per lb.	2	62½
June	6	" 15 bushels of corn, *a* 96cts. per bushel	14	40
"	"	" 3 do. rye, *a* 82cts. "	2	46
	19	" 1 cow	27	50
Aug.	21	" 1 pork ham, wt. 31⅓lbs. at 12 cents	3	76
Sept.	10	" 26 bushels potatoes, *a* 25cts.	6	50
1835				
Jan.	2	" 5 cords of oak wood, *a* $5,00	25	00
May	10	" cash to balance	2	78
			86	90

CONTRA. CR.

			$	cts
1834				
Nov.	19	By one pair boots	4	50
Dec.	12	" 3 pair boys' shoes	3	75
1835				
August	5	" cash	9	36
		This is the form of an account remaining unsettled, and open for future entries.		

FORM OF A DAY BOOK.

WILLIAM MERCHANT, N. LONDON, APRIL 1, 1834.

			$	cts
	Joseph Alberts	Dr.		
‖1	To 1 barrel flour,	$6,50		
	" 6lbs. brown Havanna sugar,	,66	7	16
	Nathaniel Curtiss	Cr.		
‖2	By 15 bushels corn a 75c per bushel,	$11,25		
	" 18¾lbs. butter a 16 cents,	1,40	12	65
	Jeremiah Goodale	Dr.		
‖3	To 5 gallons molasses a 44cts.		2	20
	Joseph Alberts	Cr.		
‖1	By 68lbs. cheese at 6½cts.		4	42
	Nathaniel Curtiss	Dr.		
‖2	To 3½ yards of broadcloth a $4,75,		16	62½
‖2	By cash,	Cr.	3	12½
	Jeremiah Goodale	Dr.		
‖3	To 38lbs. salt pork at 8½cts.	$3,23		
	" 1 pork ham 27lbs. a 12½cts.	3,37½	6	60½
	Joseph Alberts	Dr.		
‖1	To 5½ yards Irish linen at 58cts.	$3,19		
	" 1 Daboll's complete Schoolmasters' Assistant.	,38	3	57
‖1	By 2½ cords walnut wood a $6,00,	Cr.	15	00
	Nathaniel Curtiss	Dr.		
‖2	To 9 yards calico a 31 cents,	$2,79		
	" 2 brooms,	,34	3	13
	Jeremiah Goodale	Cr.		
‖3	By mending my carriage,	$3,50		
	" cash,	1,50	5	00
	Joseph Alberts	Dr.		
‖1	To 6 gallons molasses a 42 cents,	$2,52		
	" 3lbs. Hyson S. tea a 50cts.	1,50		
	" 1 silk handkerchief,	,84	4	86
	Nathaniel Curtiss	Dr.		
‖2	To 9lbs. geese feathers a 50cts.		4	50
‖2	By cash,	Cr.	3	25
	Jeremiah Goodale	Dr.		
‖3	To 5 bushels oats at 34 cents,	$1,70		
	" To cash,	2,25	3	95

Page 2.] FORM OF A DAY BOOK.

NEW LONDON, APRIL 12, 1834.

			$	cts
	Nathaniel Curtiss	Dr.		
×2	To 3¼ yards cotton sheeting *a* 16c.	$5,00		
	" 1 spool thread,	,06	5	06
	Joseph Alberts	Dr.		
×1	To 15lbs. shingle nails at 8 cents,	$1,20		
	" 1 gallon Malaga wine,	1,17	2	37
×3	Jeremiah Goodale	Cr.		
	By 3½ bushels beans *a* $1,12,		3	92
	Joseph Alberts	Dr.		
	To 9yds. satinett *a* $1,25,	$11,25		
×1	" 5 sks. sewing silk *a* 5c.	,25		
	" 2 sticks twist *a* 6 cents,	,12	11	62
×2	Nathaniel Curtiss	Cr.		
	By 2bbls. cider *a* $1,50, the barrels to be returned,		3	00
×3	Jeremiah Goodale	Dr.		
	To 3yds. bleached sheeting *a* 18cts.		0	54
×3	By cash,	Cr.	2	86

We have given an example of the Day Book sufficient to show the manner of keeping it. At the commencement of the book, the owner should write his name, together with the place of his residence, in the manner exhibited on page 1st, where the pupil will observe the name of William Merchant, who is supposed to be the person who owns the book, and residing in New London. The pupil will also observe, that the day of the month, &c. must be placed at the top of every page; and when the date alters between the top and bottom of any page, the figure denoting the day of the month is placed in a break in the lines which are drawn between the entries.

By posting an account, is meant, transferring the entries from the Day Book to the Leger. Every entry in the Day Book must be posted into the Leger; and when several articles are entered in the Day Book at one time, it is not necessary to mention them all in posting to the Leger, but merely to say "sundries," as in the following Leger.

In posting an account, begin with the first name on the Day Book. Enter it on the Leger: if it be Dr., on the left, or if Cr., on the right hand of the page, thus entering each under its proper head, placing the date and the page of the Day Book in the columns ruled for that purpose; and mark the page of the Leger on the Day Book opposite the entry.

Every Leger should have an Index, or Alphabet, to show on which page each person's account stands.

INDEX TO THE LEGER.

Page 1.] FORM OF A LEGER.

DR. JOSEPH ALBERTS. CR.

1834	day of mo.		day book	$	cts	1834			p. of day book	$	cts
Apl.	1	To sundries	1	7	16	Apl.	4	By cheese	1	4	42
"	5	" do.	1	3	57	"	5	" wood	1	15	00
"	6	" do.	1	4	86	"	15	" cash to balance		10	16
"	12	" do.	2	2	37						
"	13	" do.	2	11	62						
				29	58					29	58

Page 2.] DR. NATHANIEL CURTISS. CR.

1834				$	cts	1834				$	cts
Apl.	4	To broadcloth	1	16	62	"	1	By sundries	1	12	65
"	6	" sundries	1	3	13	"	4	" cash	1	3	12
"	10	" geese feath's	1	4	50	"	10	" cash	1	3	25
"	12	" sundries	2	5	06	"	14	" cider	2	3	00
								" balance to new account		7	29
				29	31					29	31
		To balance from old account		7	29			This is the form of an account settled but not paid in full, and the balance carried forward to commence a new account.			

Page 3.] DR. JEREMIAH GOODALE. CR.

1834				$	cts	1434				$	cts
Apl.	3	To molasses	1	2	20	Apl.	6	By sundries	1	5	00
"	5	" sundries	1	6	60	"	12	" beans	2	3	92
"	10	" sundries	1	3	95	"	16	" cash	2	2	86
"	16	" sheeting	2	0	54						
		This account, remaining unsettled, is open to receive further entries from the Day Book									

Note on Demand.

$65 $\frac{51}{100}$. *New London, October* 3, 1836

For value received, I promise to pay to ARETAS A. WILDER, or order, sixty-five dollars fifty-one cents, on demand, with interest.

ALPHA G. WILLARD.

With the words '*or order*,' a note is negotiable, that is, the person to whom it is made payable, may sell it again, provided he endorses it; and the holder may lawfully demand payment of the signer of the note; or if he refuses, or is unable to pay, of the endorser.

Note of three months.

$50 $\frac{25}{100}$. *New London, October* 3, 1837.

For value received, I promise to pay to REUBEN PRITCHARD, or bearer, fifty dollars twenty-five cents, three months after date.

CHARLES STOCKMAN.

A note made payable to A B, *or bearer*, may be collected by any person holding it, and requires no endorsement. Without the words '*or order*,' or, '*or bearer*,' a note is not negotiable.

Note at Bank.

$150. *New London, October* 3, 1836.

Ninety-five days from date, I promise to pay OLIVER OSBORN, or order, one hundred and fifty dollars, at the Union Bank, for value received.

SOLOMON PAYWELL.

Receipt for money on account.

Received of Jonathan Davidson, eleven dollars on account.
New York, October 1, 1836. DANIEL DOWNER

Receipt in full.

Received of Solomon Strong, one dollar, in full of all demands.
Norwich, October 3, 1836. JOB L. BURNHAM.

A receipt in full of all accounts cuts off accounts only, while a receipt in full of all demands, cancels all claims whatsoever.

An Order for Money.

Messrs. O'Neil & Sawney,

Pay Timothy Turner, or order, eleven dollars and fifty cents, and charge the same to my account. TITUS TRUMAN.
New London, October 3, 1836.

An Order for Goods.

Mr. B. Stark, NEW ORLEANS, Oct. 5, 1836.

Please pay the bearer one dollar in goods from your store, and charge . Your obedient servant, THOMAS PORTER.

When an order is paid, it is proper that it be receipted on the back, by the person to whom it was made payable, or if it be made payable to bearer, any person who presents it for payment may receipt it.